Modified Gravity

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Before starting with the second part of the talk, I would like to comment on the issue of conformal frames, raised yesterday by Matias Zaldarriaga.

Due to lack of time, I borrowed some transparencies from a talk by Misao Sasaki.

X-frame → W-frame

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Ginzburg Conference
on Physics
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Conformal frame independence in cosmology

Misao Sasaki

Yukawa Institute for Theoretical Physics, Kyoto University

N Makino & MS, PTP 86 (1991) 103-118.
N Deruelle & MS, arXiv:1007.3563 [gr-qc], Springer Proc in Phys 137 (2010) 247.
N Deruelle & MS, PTP Suppl 190 (2011) 143 [arXiv:1012.5386].
J Gong, J Hwang, W Park, Y Song & MS, JCAP 1109 (2011) 023 [arXiv:1107.1840]
J White, M Minamitsuji & MS, arXiv:1205.0656 [astro-ph.CO]

Conformal transformation

A few basics:

- metric and scalar curvature

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$$

$$R \to \tilde{R} = \Omega^{-2}\left[R - (D-1)\left(2\frac{\Box\Omega}{\Omega} - (D-4) g^{\mu\nu} \frac{\partial_\mu \Omega \partial_\nu \Omega}{\Omega^2} \right)\right]$$

- matter fields (for $D = 4$)

$$\phi \;\to\; \tilde{\phi} = \Omega^{-(D-2)/2}\phi \quad \left(= \Omega^{-2}\phi \right) \qquad \text{scalar}$$

$$A_\mu \to \tilde{A}_\mu = \Omega^{-(D-4)/2} A_\mu \quad \left(= A_\mu \right) \qquad \text{vector}$$

$$\psi \;\to \tilde{\psi} = \Omega^{-(D-1)/2}\psi \quad \left(= \Omega^{-3/2}\psi \right) \qquad \text{fermion}$$

2. Standard ("baryonic") action in 4D

(for Universe at T $\lesssim$ GeV)

'Jordan' frame (= matter minimally coupled to gravity)

$$S = \int d^4x \sqrt{-g} \left[-i \bar{\psi}_X \gamma^\mu \left(\overline{D}_\mu - i e_X A_\mu \right) \psi_X - m_X \bar{\psi}_X \psi_X - \frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + \cdots \right]$$

$$\bar{\psi}\gamma^\mu \overleftrightarrow{D}_\mu \psi = \frac{1}{2}\left[\bar{\psi}\gamma^\mu D_\mu \psi - (D_\mu \bar{\psi})\gamma^\mu \psi\right] ,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad D_\mu = \partial_\mu - \frac{1}{4}\omega_{ab\mu}\Sigma^{ab} ,$$

$$\Sigma^{ab} = \frac{1}{2}\left[\gamma^a, \gamma^b\right] , \quad \omega_{ab\mu} = e_{a\nu} \nabla_\mu e^\nu_b .$$

ψ_X : X = electron/proton/...

A : electromagnetic 4-potential

Effect of conformal transformation

For $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$

$$S = \int d^4x \sqrt{-\tilde{g}} \left[i\bar{\tilde{\psi}}\,\tilde{\gamma}^\mu \left(\overline{\tilde{D}}_\mu - ieA_\mu \right) \tilde{\psi} - \tilde{m}\,\bar{\tilde{\psi}}\tilde{\psi} - \frac{1}{4}\tilde{g}^{\mu\alpha}\tilde{g}^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + \cdots \right]$$

where $\tilde{\gamma}^\mu = \Omega^{-1}\gamma^\mu, \quad \tilde{\psi} = \Omega^{-3/2}\psi, \quad \tilde{m} = \Omega^{-1}m$.

(A_μ is conformal invariant in 4 dim)

Conformal transformation from `Jordan frame' to any other frame results in spacetime-dependent mass.

And this is the only effect, provided dynamics of Ω (at short distances) can be neglected.

(Ω may be dynamical on cosmological scales)

Conformal transformation:

$$ds^2 \to d\tilde{s}^2 = \Omega^2 ds^2; \quad \Omega = \frac{1}{a}$$

$$\Rightarrow \; d\tilde{s}^2 = -d\eta^2 + d\sigma_{(3)}^2; \quad d\eta = \frac{dt}{a(t)}$$

In this conformal frame, the universe is static.

no Hubble flow.

photons do not redshift...

Is this frame unphysical?

In this static frame,

- electron mass varies in time: $\tilde{m}(\eta) = m\,\Omega^{-1} = \dfrac{m}{1+z}$

  where "z" is defined by

$$1 + z \equiv \Omega = \frac{1}{a(\eta)} \quad \left(a_0 = a(\eta_0) = 1\right)$$

- Bohr radius $\propto m^{-1}$ ⇔ atomic energy levels $\propto m$:

energy level in 'static' frame

$$\tilde{E}_n = \frac{E_n}{1+z}$$

energy level in 'Jordan' frame

Thus frequency of photons emitted from a level transition $n \to n'$ at time $z = z(\eta)$ is

$$\tilde{E}_{nn'} = \frac{E_{nn'}}{1+z}$$

this is exactly what we observe as Hubble's law!

Conformal-Frame (In)dependence of Cosmological Observations in Scalar-Tensor Theory

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(Dated: October 24, 2013)

We provide the correspondence between the variables in the Jordan frame and those the Einstein frame in scalar-tensor gravity and consider the frame-(in)dependence of the cosmological observables. In particular, we show that the cosmological observables/relations (redshift, luminosity distance, temperature anisotropies) are frame-independent. We also study the frame-dependence of curvature perturbations and find that the curvature perturbations are conformal invariant if the perturbation is adiabatic and the entropy perturbation between matter and the Brans-Dicke scalar is vanishing. The relation among various definitions of curvature perturbations in the both frames is also discussed, and the condition for the equivalence is clarified.

JCAP 1310:040,2013

Summary Part II

- How to distinguish between MG theory and the ΛCDM model?
- Perturbations: Newtonian, MG.
- Observable quantities.

How to distinguish between MG theory and the ΛCDM model?

Background evolution does not help, since for a given a *(t)* , it is possible to find a potential *V(*φ*)* (DE) or a family of *f(R)* theories that has such a(t) as a solution, see Basset et al (2002) for DE, and Pogossian and Silvestri (2008), for f(R).

This degeneracy can be broken by considering observables determined by structure formation. Hence we must look at perturbations, or fully nonlinear phenomena.

Perturbations - Newtonian theory

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \boldsymbol{v} = 0,$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} + \frac{1}{\rho}\nabla p + \nabla \varphi = 0,$$

$$\nabla^2 \varphi - 4\pi G \rho = 0.$$

$$\frac{\partial s}{\partial t} + \boldsymbol{v} \cdot \nabla s = 0.$$

(no dissipation)

Solution:

$$\rho = \rho_0 \left(\frac{a_0}{a}\right)^3,$$

$$\boldsymbol{v} = \frac{\dot{a}}{a}\boldsymbol{r},$$

$$\varphi = \tfrac{2}{3}\pi G \rho r^2,$$

$$p = p(\rho, S),$$

$$s = \text{const.};$$

$$\boldsymbol{r} = \boldsymbol{r}_0 \frac{a}{a_0}.$$ **(r_0 is a comoving radius)**

Describes the expansion or contraction of an isotropic and homogeneous distribution of matter.

Perturbations:

$$\rho = \rho_0 + \delta\rho, \boldsymbol{v} = \delta\boldsymbol{v}, p = p_0 + \delta p,$$

$$s = s_0 + \delta s, \qquad \varphi = \varphi_0 + \delta\varphi.$$

Linearizing the EOM:

$$\dot{\delta\rho} + 3\frac{\dot{a}}{a}\delta\rho + \frac{\dot{a}}{a}(\boldsymbol{r} \cdot \nabla)\delta\rho + \rho(\nabla \cdot \delta\boldsymbol{v}) = 0,$$

$$\dot{\delta\boldsymbol{v}} + \frac{\dot{a}}{a}\delta\boldsymbol{v} + \frac{\dot{a}}{a}(\boldsymbol{r} \cdot \nabla)\delta\boldsymbol{v} = -\frac{1}{\rho}\nabla\delta p - \nabla\delta\varphi,$$

$$\boxed{\nabla^2\delta\varphi - 4\pi G\delta\rho = 0,}$$

$$\dot{\delta s} + \frac{\dot{a}}{a}(\boldsymbol{r} \cdot \nabla)\delta s = 0,$$

$$\delta u_i = u_i(t)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}),$$

(i = 1 ,2 , 3 , 4 ➡ D, <u>V</u> , Φ, Σ).

$$v_s^2 = (\partial p/\partial\rho)_S$$

$$\dot{D} + 3\frac{\dot{a}}{a}D + \mathrm{i}\rho\boldsymbol{k}\cdot\boldsymbol{V} = 0,$$

$$\dot{\boldsymbol{V}} + \frac{\dot{a}}{a}\boldsymbol{V} + \mathrm{i}v_s^2\boldsymbol{k}\frac{D}{\rho} + \mathrm{i}\frac{\boldsymbol{k}}{\rho}\left(\frac{\partial p}{\partial s}\right)_\rho\Sigma + \mathrm{i}\boldsymbol{k}\Phi = 0,$$

$$k^2\Phi + 4\pi GD = 0,$$

$$\dot{\Sigma} = 0.$$

For Σ = 0 and <u>V</u> parallel to *k* ,

$$\dot{D} + 3\frac{\dot{a}}{a}D + \mathrm{i}\rho kV = 0,$$

$$\dot{V} + \frac{\dot{a}}{a}V + \mathrm{i}k\left(v_s^2 - \frac{4\pi G\rho}{k^2}\right)\frac{D}{\rho} = 0.$$

Defining $D = \rho\,\delta$,

$$\dot{\delta} + \mathrm{i}kV = 0, \quad \rightarrow \quad \ddot{\delta} + \mathrm{i}k\left(\dot{V} - \frac{\dot{a}}{a}V\right) = 0.$$

Replacing V and V• ,

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + (v_{\mathrm{s}}^2 k^2 - 4\pi G\rho)\delta = 0,$$

$$\Phi = -4\pi G\rho\frac{\delta}{k^2}$$

The behaviour of the sols. depends of

$$\lambda_{\mathrm{J}} \simeq v_{\mathrm{s}}\left(\frac{\pi}{G\rho}\right)^{1/2},$$

For $\lambda \ll \lambda_J$, the sols. are oscillatory.
For $\lambda \gg \lambda_J$, there are growing and decaying solutions.
Note that λ_J depends of t through ρ.

Background:

$$\rho = \frac{1}{6\pi G t^2},$$

$$a = a_0 \left(\frac{t}{t_0}\right)^{2/3},$$

$$\frac{\dot{a}}{a} = \frac{2}{3t},$$

$$v_{\mathrm{s}} = \left(\frac{5 k_{\mathrm{B}} T_{\mathrm{m}}}{3m}\right)^{1/2} = \left(\frac{5 k_{\mathrm{B}} T_{0\mathrm{m}}}{3m}\right)^{1/2} \frac{a_0}{a}.$$

(Monoatomic gas, mass m).

$$\ddot{\delta} + \frac{4}{3}\frac{\dot{\delta}}{t} - \frac{2}{3t^2}\left(1 - \frac{v_{\mathrm{s}}^2 k^2}{4\pi G \rho}\right)\delta = 0.$$

Hence, for small enough k,

$$\frac{\delta\rho}{\rho} \propto t^{-[1 \pm 5(1 - 6 v_{\mathrm{s}}^2 k^2 / 25\pi G \rho)^{1/2}]/6} \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{r}).$$

$$\lambda > \lambda_{\mathrm{J}}' = \frac{\sqrt{24}}{5} v_{\mathrm{s}} \left(\frac{\pi}{G\rho}\right)^{1/2},$$

Growing solution ➡ Instability.

The equation for the density contrast,

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + (v_s^2 k^2 - 4\pi G\rho)\delta = 0,$$

has solutions of the form

$$\delta(t, x) = D_+(t)\epsilon_+(x) + D_-(t)\epsilon_-(x)$$

where the ϵ are the initial density fields.

*** We shall see that in the case of GR, the dependence of the equation for δ with *H* is maintained.**

*** A useful quantity is the growth factor, given by (in the case of the standard model)**

$$f(\Omega_{m0}, \Omega_{\Lambda 0}) = \frac{d \ln D_+}{da}$$

The calculation of perturbations in GR and MG theories has some features alike to the Newtonian case, but there are additional difficulties. To name a few:

1) **There are three types of perturbations of the metric (scalars, vector, and tensor), which are decoupled to first order. Scalar perturbations may lead to growing inhomogeneities, while the tensor perturbations lead to gravitational waves (vector perts. decay with the expansion).**

2) **One needs to keep only real perturbations, discarding those that are gauge artifacts (i.e., can be eliminated by coordinate transformations). Only gauge-invariant quantities are relevant.**

3) **In the case of MG theories, it is necessary to perturb also the extra degrees of freedom.**

Matter density perturbations in metric *f(R)* theories

De Felice y Tsujikawa (2010)

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Psi)\delta_{ij}dx^i dx^j\,,$$

After a long calculation under the assumption of several hypotheses,

$$\frac{d^2\delta_m}{dN^2} + \left(\frac{1}{2} - \frac{3}{2}w_{\rm eff}\right)\frac{d\delta_m}{dN} - \frac{3}{2}\Omega_m \frac{4+3M^2a^2/k^2}{3(1+M^2a^2/k^2)} = 0\,,$$

$$\delta_m \simeq \delta\rho_m/\rho_m.$$

$$F = \frac{df}{dR}$$

$N = \ln a$, $w_{\rm eff} = -1 - 2\dot{H}/(3H^2)$, and $\Omega_m = 8\pi G\rho_m/(3FH^2)$

$$M^2 = \frac{1}{3}\left(\frac{F}{F_{,R}} - R\right)$$

In the regime $M^2 \gg k^2/a^2$, this equation reduces to the one in GR, while in the opposite regime they are solutions are non-standard. This has consequences on the growth factor.

From the EOM for the perturbations we also get

$$\frac{k^2}{a^2}\Psi \simeq -\frac{\kappa^2\delta\rho_m}{2F}\frac{2+3M^2a^2/k^2}{3(1+M^2a^2/k^2)}, \qquad \frac{k^2}{a^2}\Phi \simeq -\frac{\kappa^2\delta\rho_m}{2F}\frac{4+3M^2a^2/k^2}{3(1+M^2a^2/k^2)}$$

while in GR, $\Phi = \Psi$

A useful quantity is

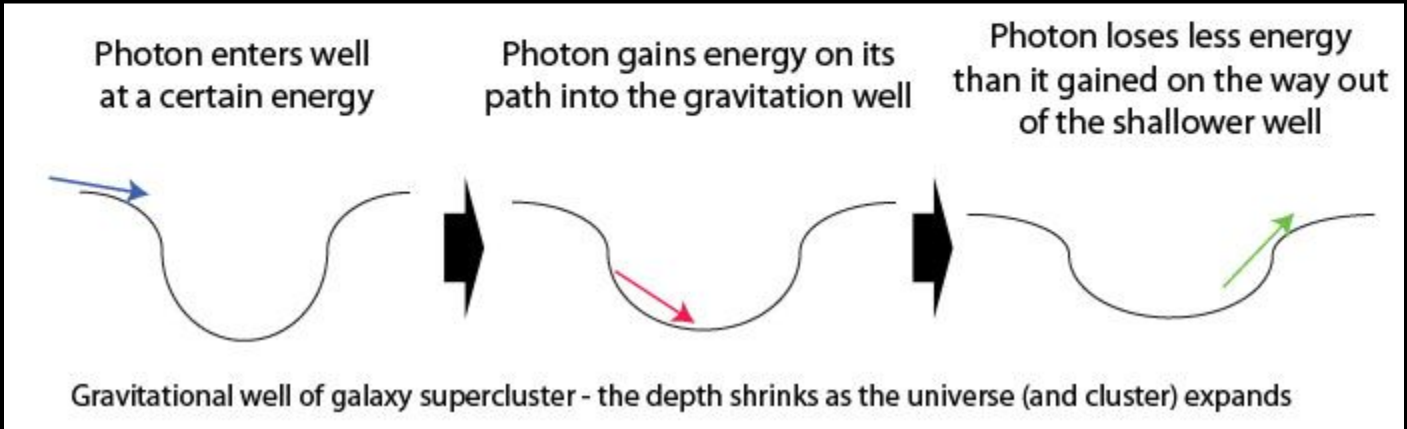
$$\Phi_{\rm eff} \equiv (\Phi+\Psi)/2\,. \qquad \Phi_{\rm eff} \simeq -\frac{\kappa^2}{2F}\frac{a^2}{k^2}\delta\rho_m\,.$$

So in principle, for a given *f(R)*, we can solve the diff. eq. for δ_m as a function of a, and then calculate the potentials.

Note that all these quantities are scale-dependent (through *k*), due to M.

In which observables can we see the difference between ΛCDM and *f(R)* theories?

- **Integrated Sachs-Wolfe effect**



$$\left.\frac{\Delta T}{T}\right|_{\mathrm{ISW}} = -\int \frac{\mathrm{d}(\Psi + \Phi)}{\mathrm{d}t} \frac{a(z)\mathrm{d}z}{H(z)} \,.$$

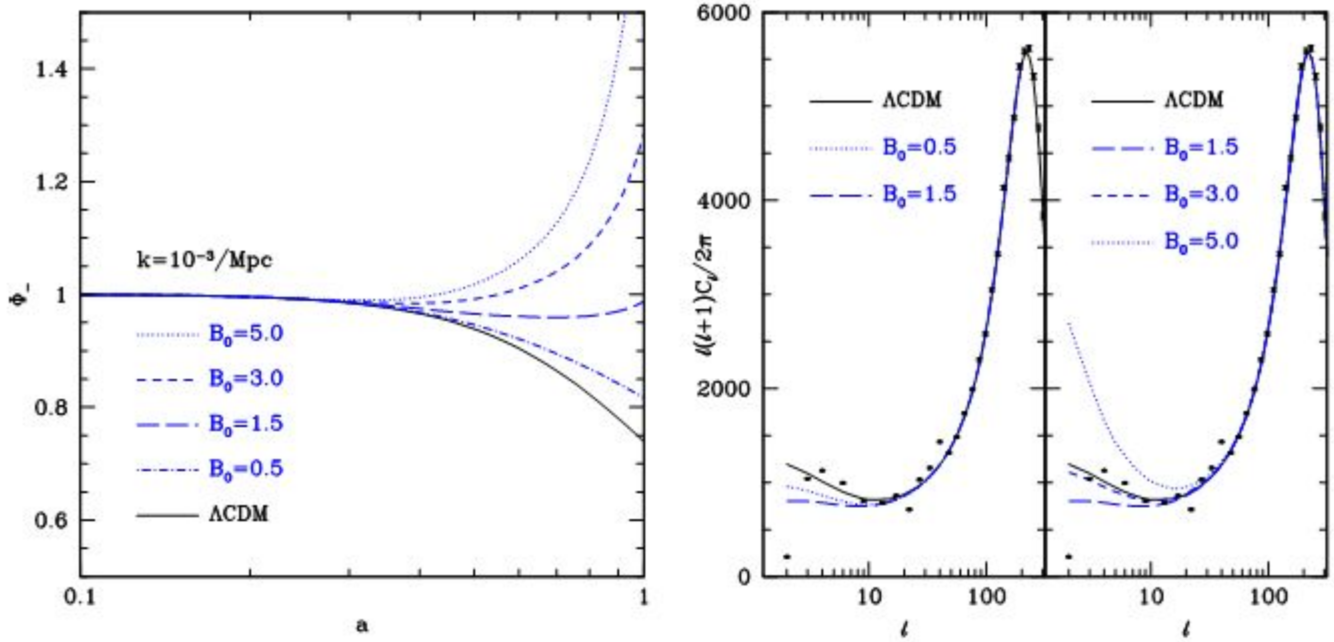


Figure 7: (Left) Evolution of the effective gravitational potential $\Phi_{\rm eff}$ (denoted as Φ_- in the figure) versus the scale factor a (with the present value $a = 1$) on the scale $k^{-1} = 10^3$ Mpc for the ΛCDM model and $f(R)$ models with $B_0 = 0.5, 1.5, 3.0, 5.0$. As the parameter B_0 increases, the decay of $\Phi_{\rm eff}$ decreases and then turns into growth for $B_0 \gtrsim 1.5$. (Right) The CMB power spectrum $\ell(\ell+1)C_\ell/(2\pi)$ for the ΛCDM model and $f(R)$ models with $B_0 = 0.5, 1.5, 3.0, 5.0$. As B_0 increases, the ISW contributions to low multipoles decrease, reach the minimum around $B_0 = 1.5$, and then increase. The black points correspond to the WMAP 3-year data [25]. From Ref. [85].

$$B = \frac{f_{RR}}{1+f_R} R' \frac{H}{H'}$$

evaluated today characterizes the f(R) theory that has the expansion of the ΛCDM model.

In which observables can we see the difference between ΛCDM and *f(R)* theories? (For a review see Joyce et al, 2016)

- **Growth factor:**

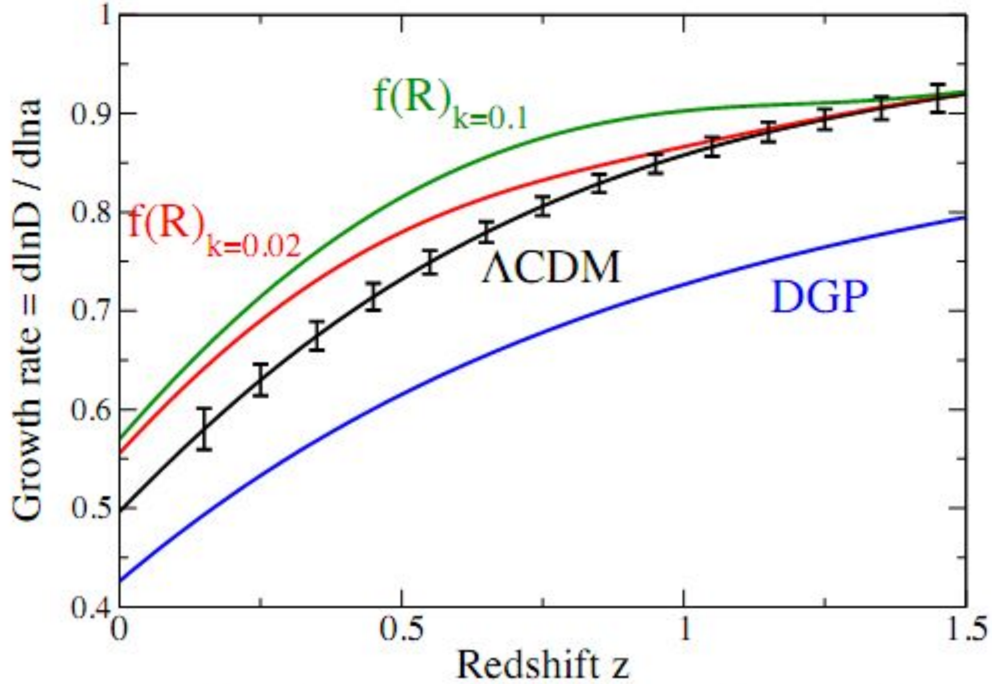


Figure 2. *Constraints on the growth of density fluctuations in the Universe with errors projected from a future survey designed with DESI specifications. The curves show the derivative of the logarithmic growth with respect to the logarithmic scale factor — a quantity readily measured from the clustering of galaxies in redshift space — as a function of redshift. We show theory predictions for the ΛCDM model, as well as for two modified-gravity models: the Dvali-Gabadadze-Porrati braneworld model [3] and the $f(R)$ modification to the Einstein action [4]. Because growth in the $f(R)$ models is generically scale-dependent, we show predictions at two wavenumbers, $k = 0.02\,h\,\mathrm{Mpc}^{-1}$ and $k = 0.1\,h\,\mathrm{Mpc}^{-1}$. LSST projects to impose constraints of similar excellent quality on the growth function $D(a)$.*

$$f(R) = -c\,r\,(1 - e^{-R/r})\,.$$

Linder (2009)

Huterer et al (2015)

Other theories:

- Einstein-æther theories, Jacobson & Speranza, PRD 92, 044030 (2015).
- TeVes, Saridakis, C&Q Grav., 26, 143001 (2009).
- Bi-metric theories, Schmidt-May & Von Strauss, 1512.00021 [hep-th].
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- *f(T),* Cai et al, Rept. Prog. Phys., 79, 10, 106901 (2016).
- Hybrid *f (R,* $\mathscr{R}$ *)* theories, S. Capozziello et al, Universe 1, 188 (2016).
- “Brane worlds”, Maartens & Koyama, Living Rev. Rel. 13, 5 (2010).
- ...

Related topics:

- **Screening mechanism: the corrections to Newton´s law (coming from the extra degrees of freedom) must be suppressed to be in accordance with the results of lab experiments (high density environment) without spoiling the modifications for cosmology (very low densities). There are several ways to do this in ST theories (symmetron, chameleon, Vainshtein), using the freedom in the arbitrary functions. Some of these may be translated to f(R) theories.**

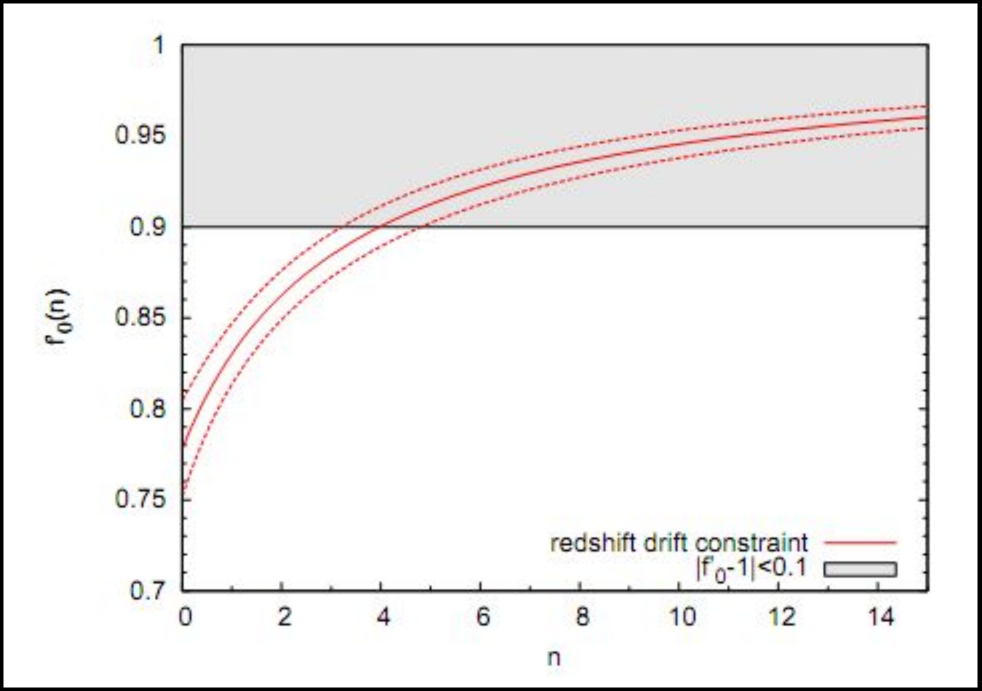
- **Limits on a given f(R).**

  Limits on the parameters entering a given f(R) can be set from observations using Cosmography, where observables are expressed as a power series of the redshift, with coeffs. that depend of the measurable quantities H_0, q_0, j_0, ... This kinematical series can be compared to the one obtained using the dynamics of the given f(R), thus obtaining limits on the parameters of the theory. Ex.: energy conditions (SEPB, 2006), redshift drift:

$$\frac{\Delta z}{\Delta t_0} = (1+z)H_0 - H(z).$$

$$\frac{\Delta z}{\Delta t_0}(z) = -H_0 q_0 z + \frac{1}{2}H_0\left(q_0^2 - j_0\right)z^2 + \frac{1}{2}H_0\left[\frac{1}{3}\left(s_0 + 4q_0 j_0\right) + j_0 - q_0^2 - q_0^3\right]z^3 + \mathcal{O}(z^4).$$

$$\begin{aligned}\frac{\Delta z}{\Delta t_0}(z) \;=\; & \Bigg\{\left[6q_0 H_0^2 f_0' + f_0 - 6\Omega_{m,0}H_0^2\right](j_0 - q_0 - 2)^2\frac{f_0'''}{6f_0''} + \\ & + H_0^2 f_0''\left[(j_0-2)^2 - q_0(3q_0 + (q_0 - j_0)^2 - 2j_0)\right] + f_0'[q_0(q_0^2 + 6q_0 + 2j_0 + s_0) + 2j_0 + 4] \\ & + \frac{f_0}{36H_0^2}(-s_0 - q_0^2 - 6 - 8q_0) + \frac{\Omega_{m,0}}{6}(s_0 + 3j_0 + 5q_0 + q_0^2)\Bigg\}\frac{z}{f_0'' H_0(j_0 - q_0 - 2)^2} + \mathcal{O}(z^2).\end{aligned}$$



Limits on the parameters of the Hu & Sawicky theory using the RD, Teppa Pannia & SEPB (2013).