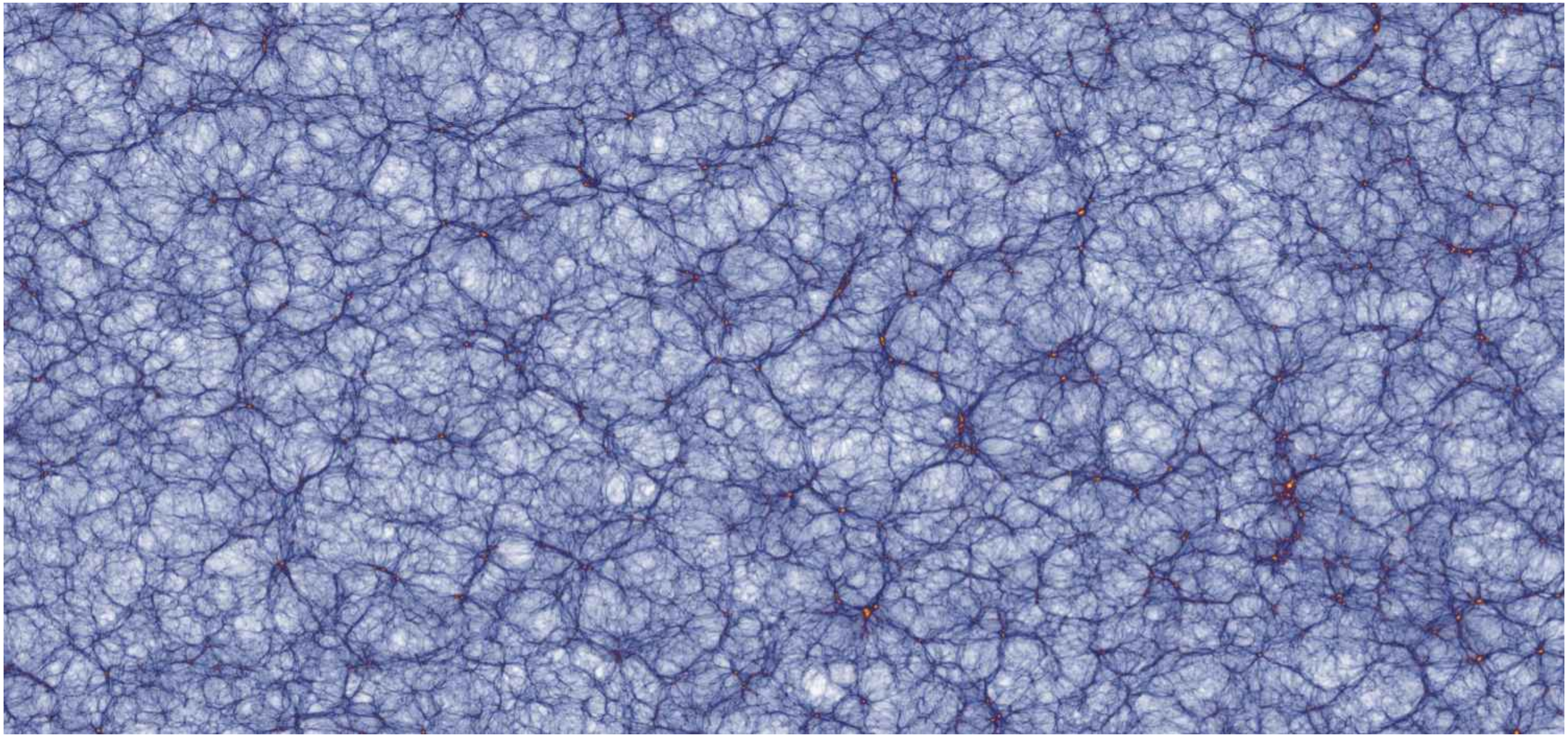
Simulating the formation of structure in the Universe – *Lecture 2*



Raul E. Angulo

- → Background
- → Methods
- → **Current State of the Art**
- → The next decade
- → Open questions & challenges

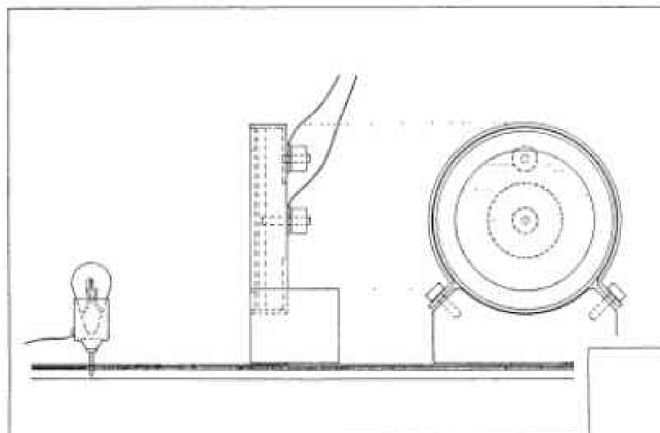
First N-body calculation: light bulbs, photocells and galvanometers

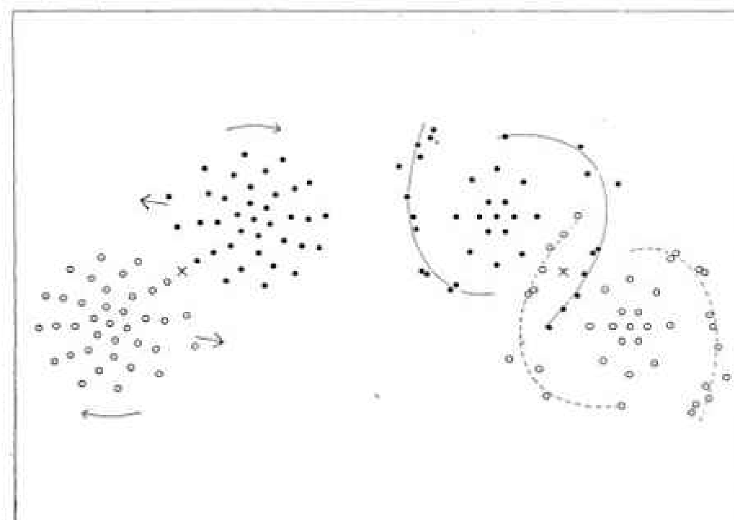
VOLUME 94 NOVEMBER 1941 NUMBER 3

ON THE CLUSTERING TENDENCIES AMONG THE NEBULAE

II. A STUDY OF ENCOUNTERS BETWEEN LABORATORY MODELS OF STELLAR SYSTEMS BY A NEW INTEGRATION PROCEDURE

ERIK HOLMBERG



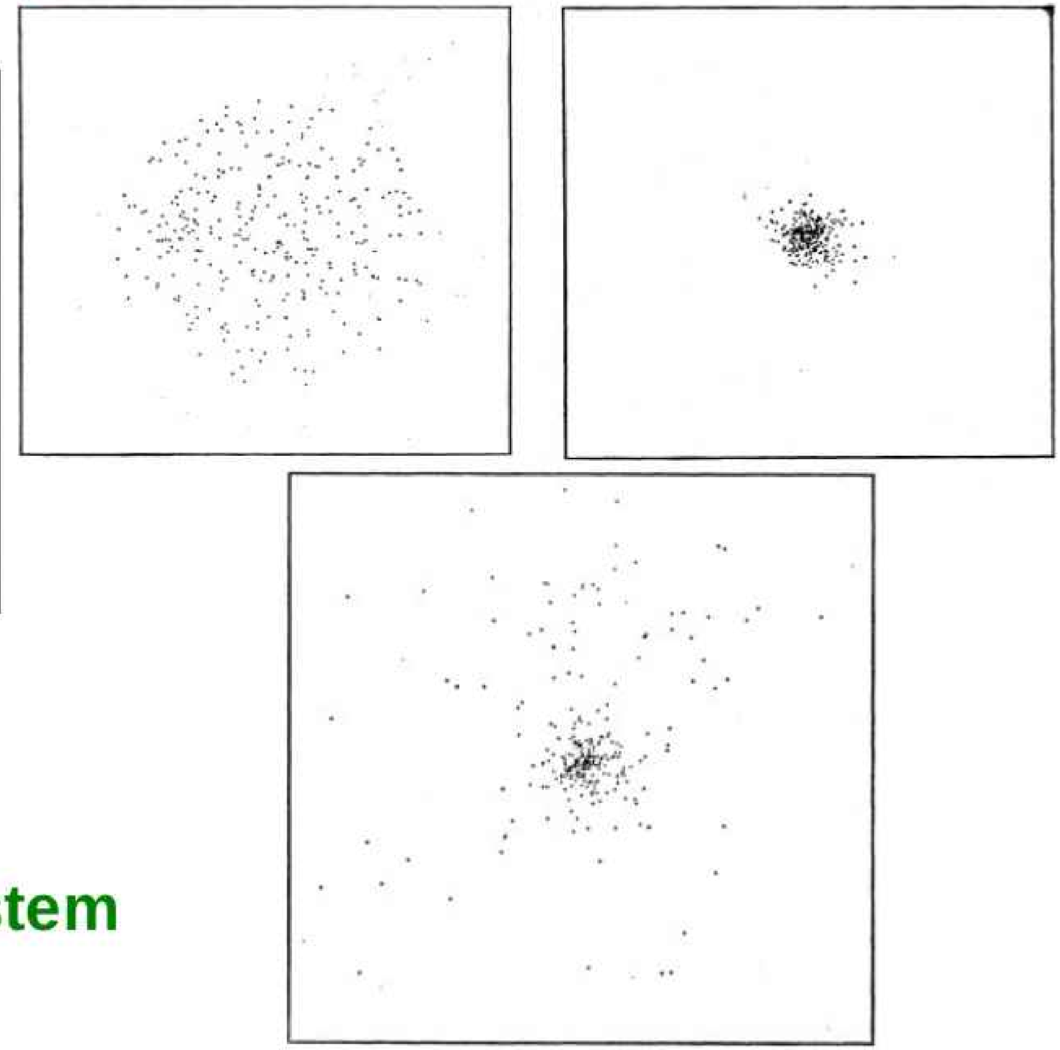


Tidal features appear in interacting nebulae

N-body calculation supports the idea of dark Matter

A rotating group of 300 bodies ends up *too* concentrated.

1970-1974, Princeton



(c)

Dark matter (M/L=10) is needed to stabilize the system

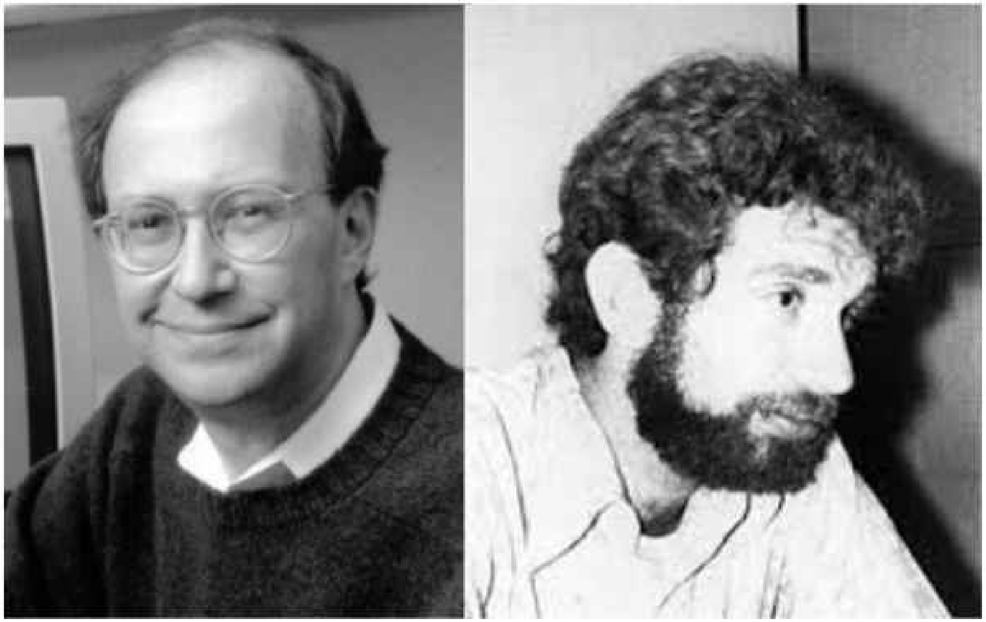
First “cosmological” N-body simulation

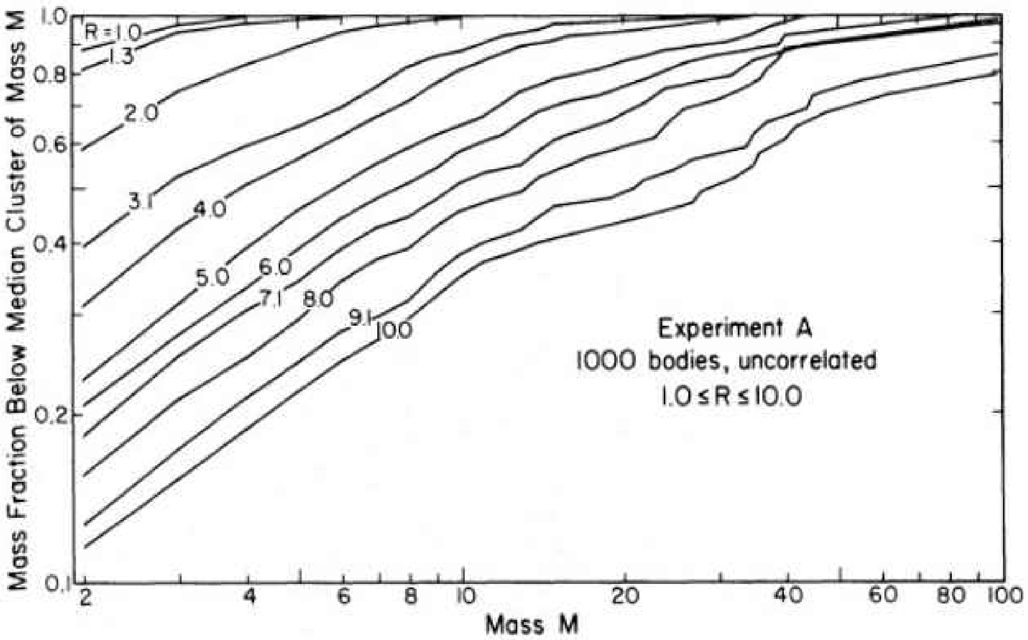
FORMATION OF GALAXIES AND CLUSTERS OF GALAXIES BY SELF-SIMILAR GRAVITATIONAL CONDENSATION*

WILLIAM H. PRESS AND PAUL SCHECHTER
California Institute of Technology
Received 1973 August 1

Press-Schchter formalism

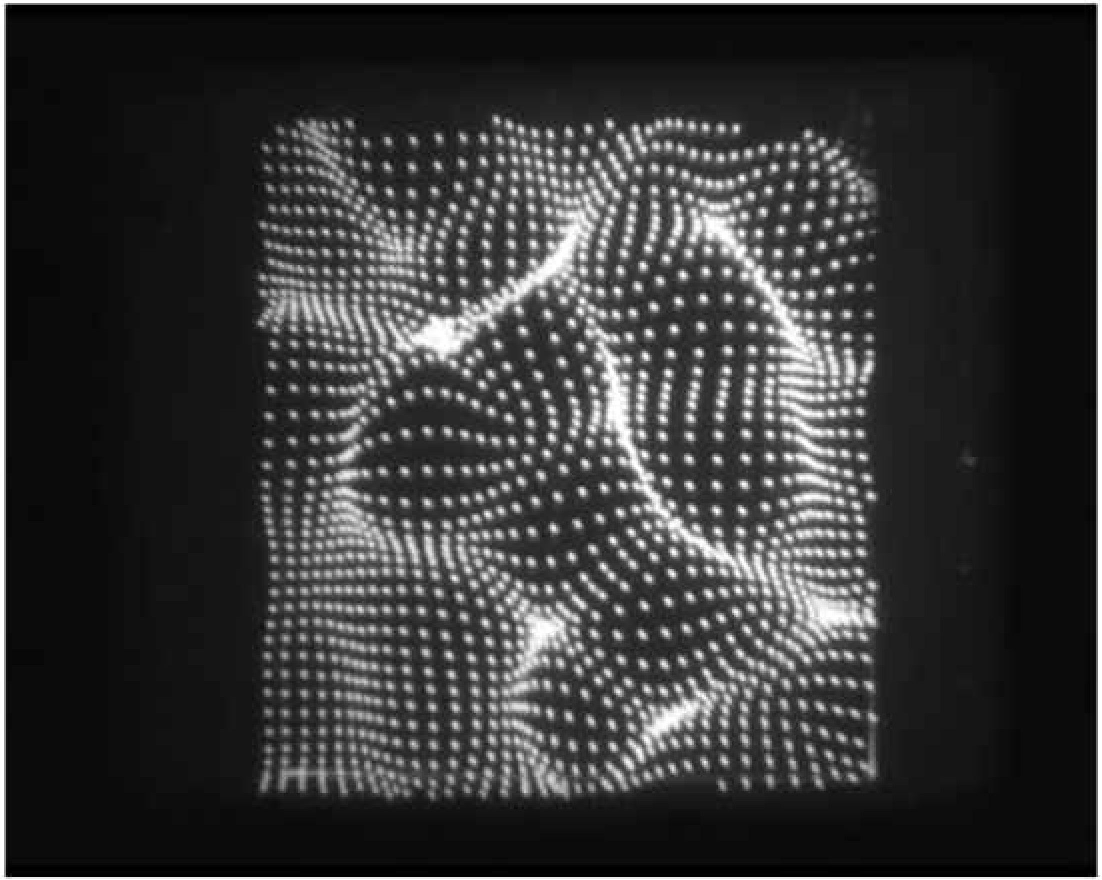
$$p(\delta, V) = (2\pi)^{-1/2}\delta_*^{-1} \exp\left[-\tfrac{1}{2}\delta^2/\delta_*^2\right],$$





Nonlinear structure formation from Gaussian initial conditions in an expanding universe

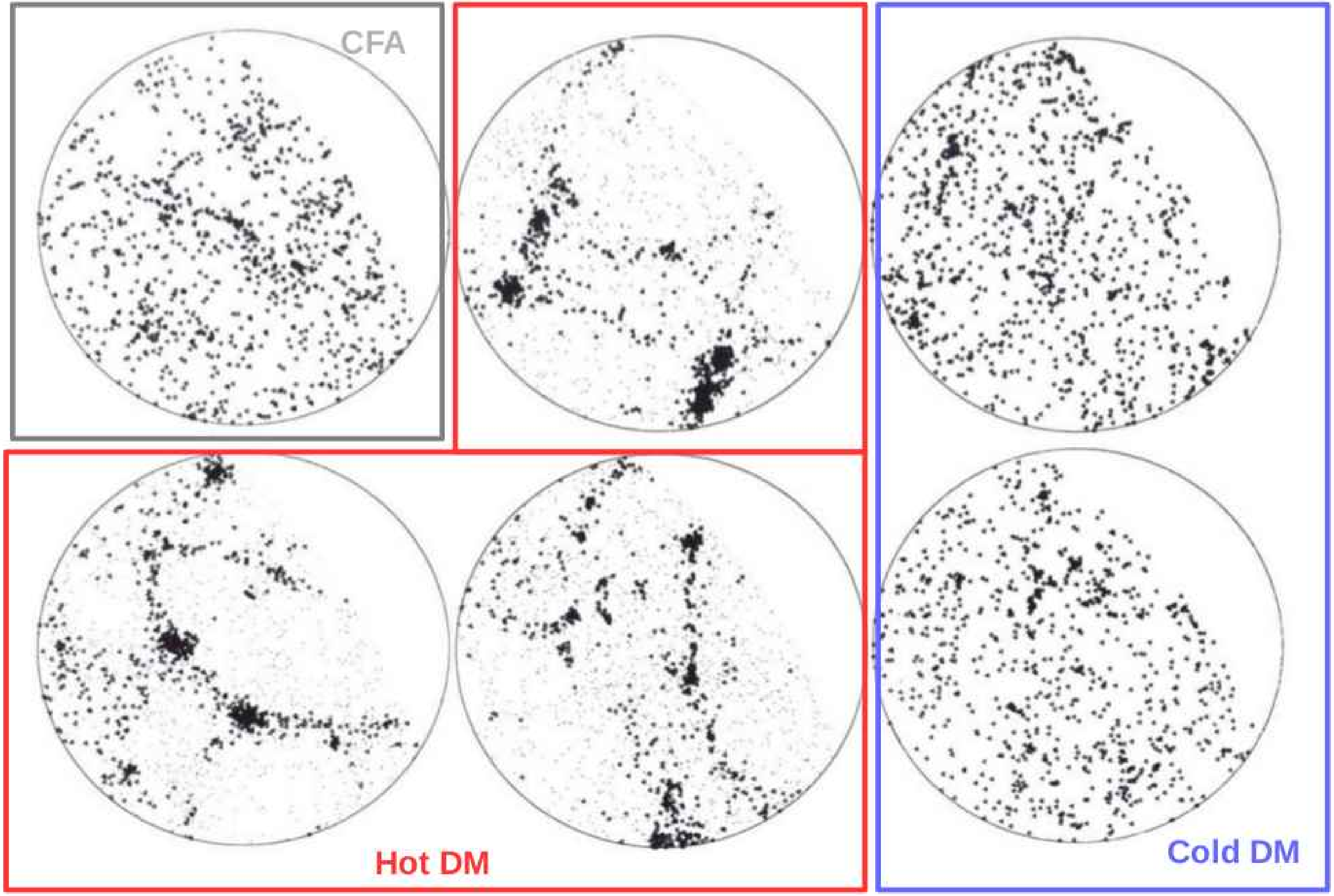
1977: First numerical simulations of the evolution of gravitational instabilities



A. G. Doroshkevich, E. V. Kotok, S. F. Shandarin **1977**:
B. "Evolution of the Density Field according to the Theory of Gravitational Instability

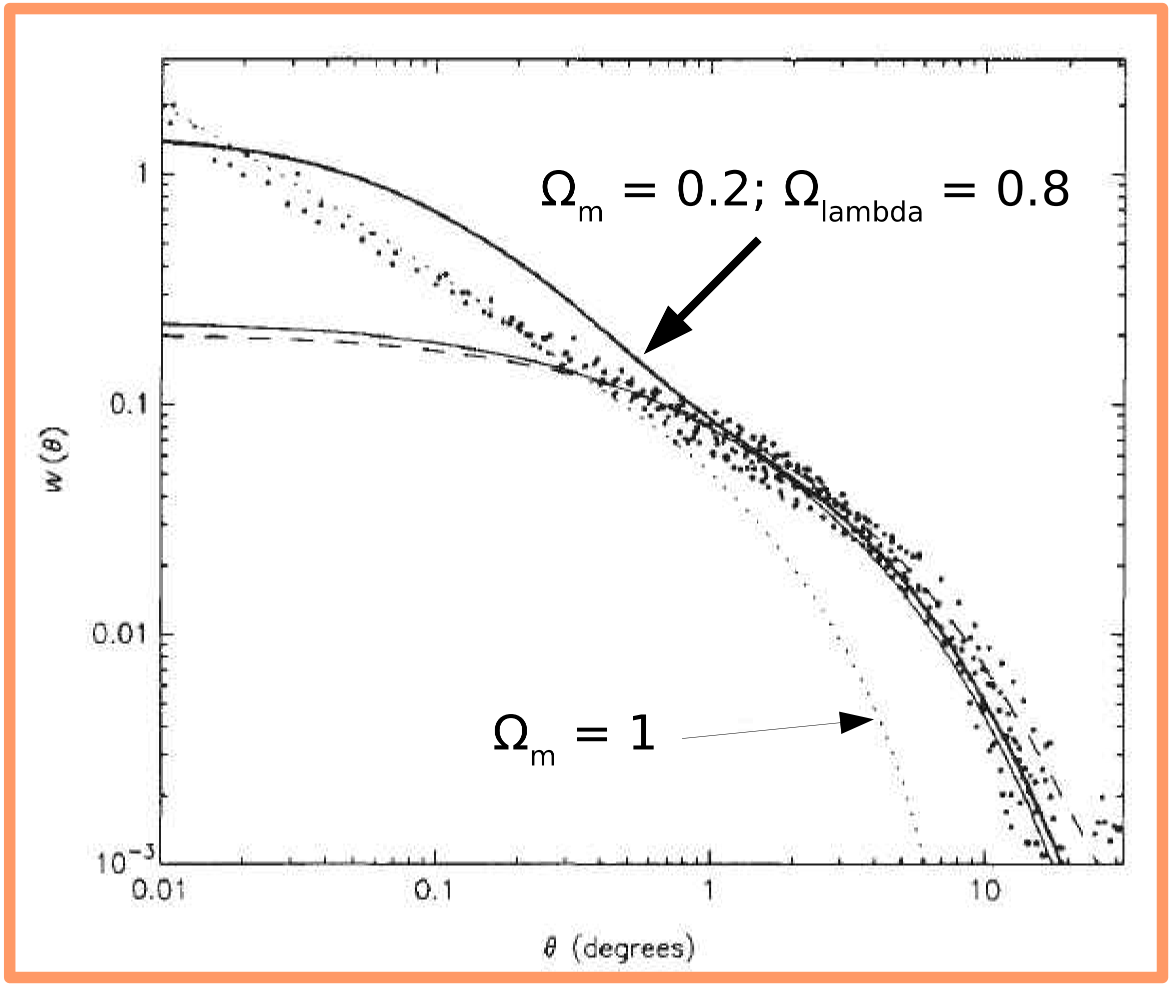
From Jaan Einasto and Tartu university

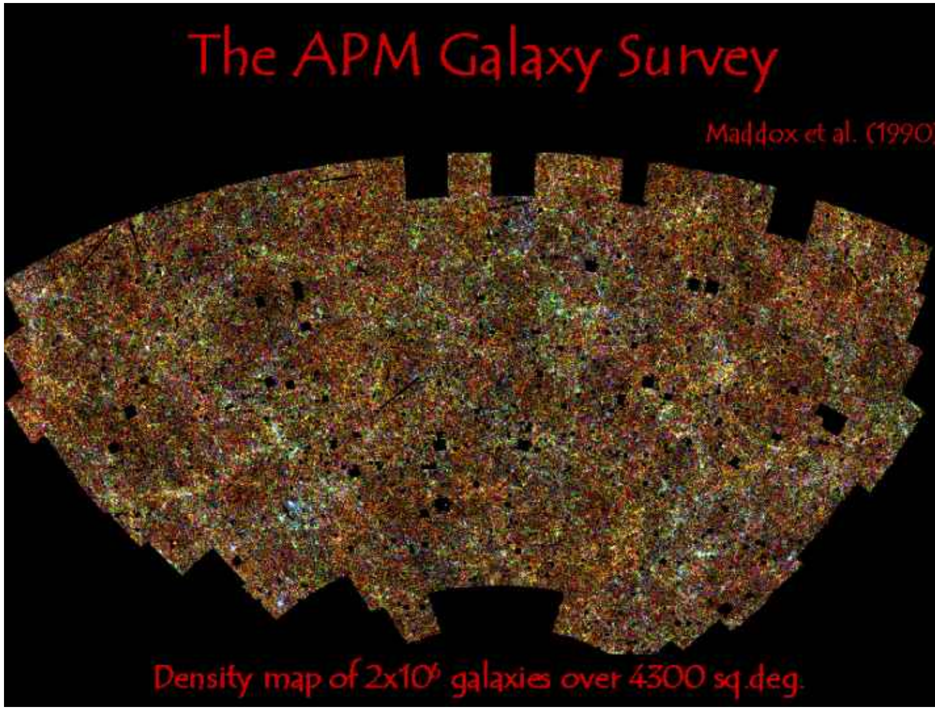
1985: The CDM model plus gravitational instability can explain qualitatively the observed universe



Davis, Efstathiou, Frenk & White 1985

1990: A cosmological constant is needed to explain the observed clustering of galaxies



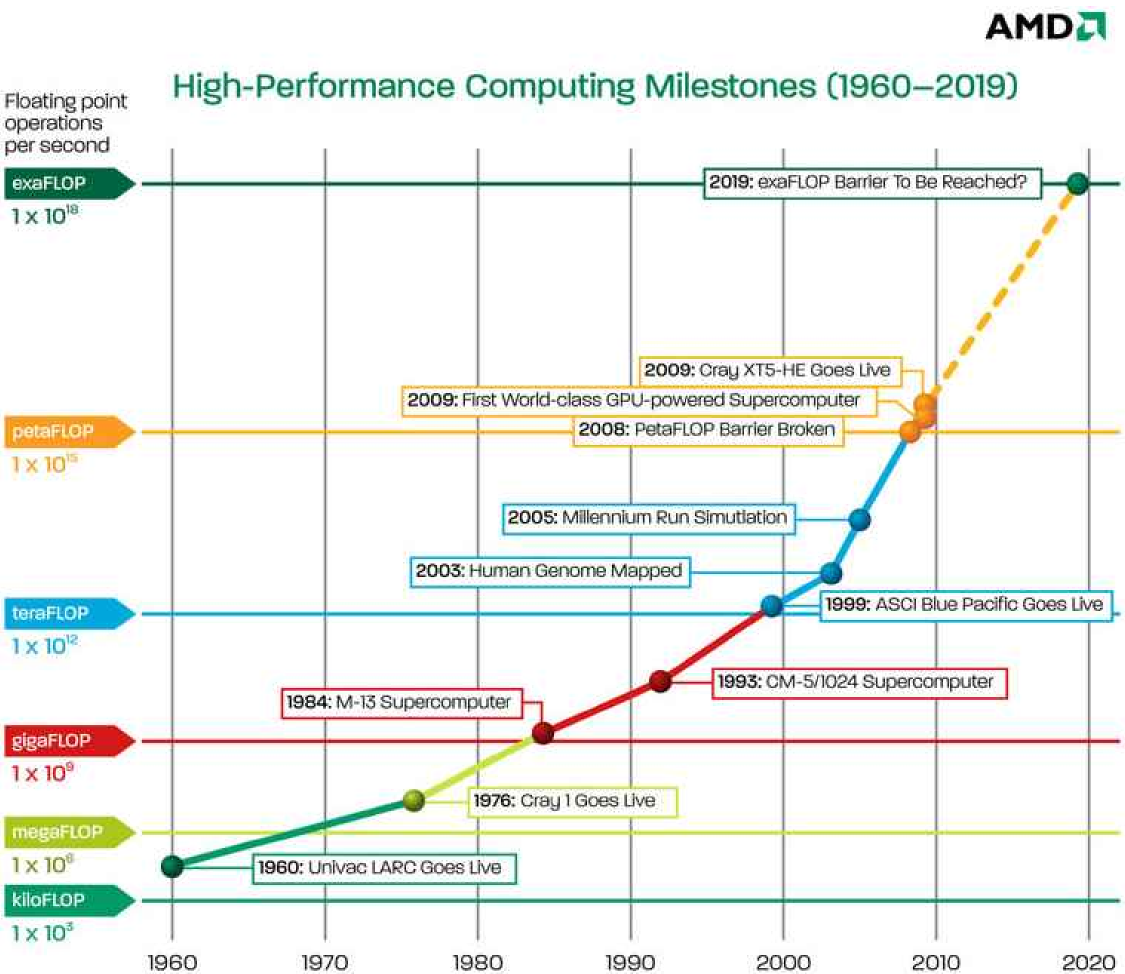


"We argue that the successes of the CDM theory can be retained and the new observation accommodated in a spatially flat cosmology in which as much as 80% of the critical density is provided by a positive cosmological constant..."

Efsthathiou, Sutherland & Maddox (1990)

Exponential Growth of Computing Power

Exascale supercomputing and 10-trillion particle Simulations are expected to be reached by 2020



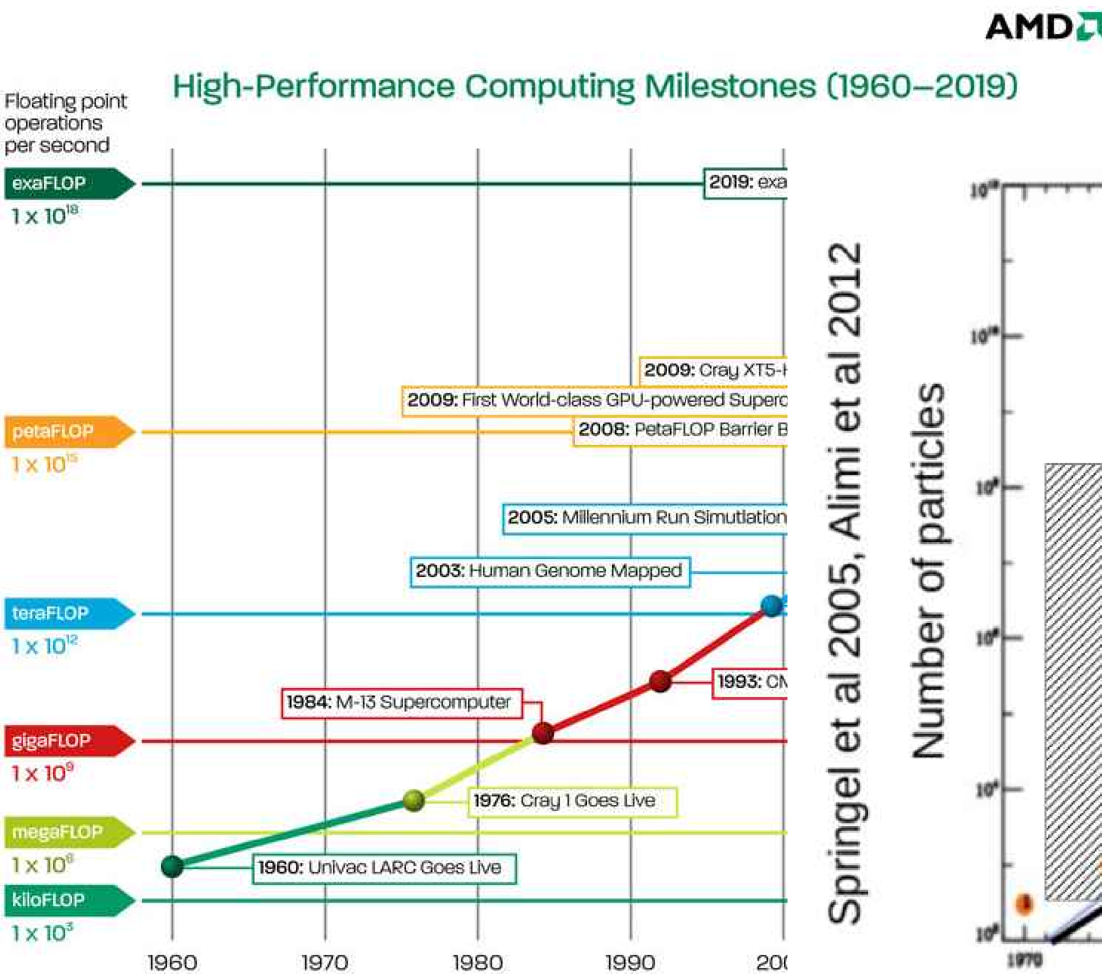
Sunway TaihuLight

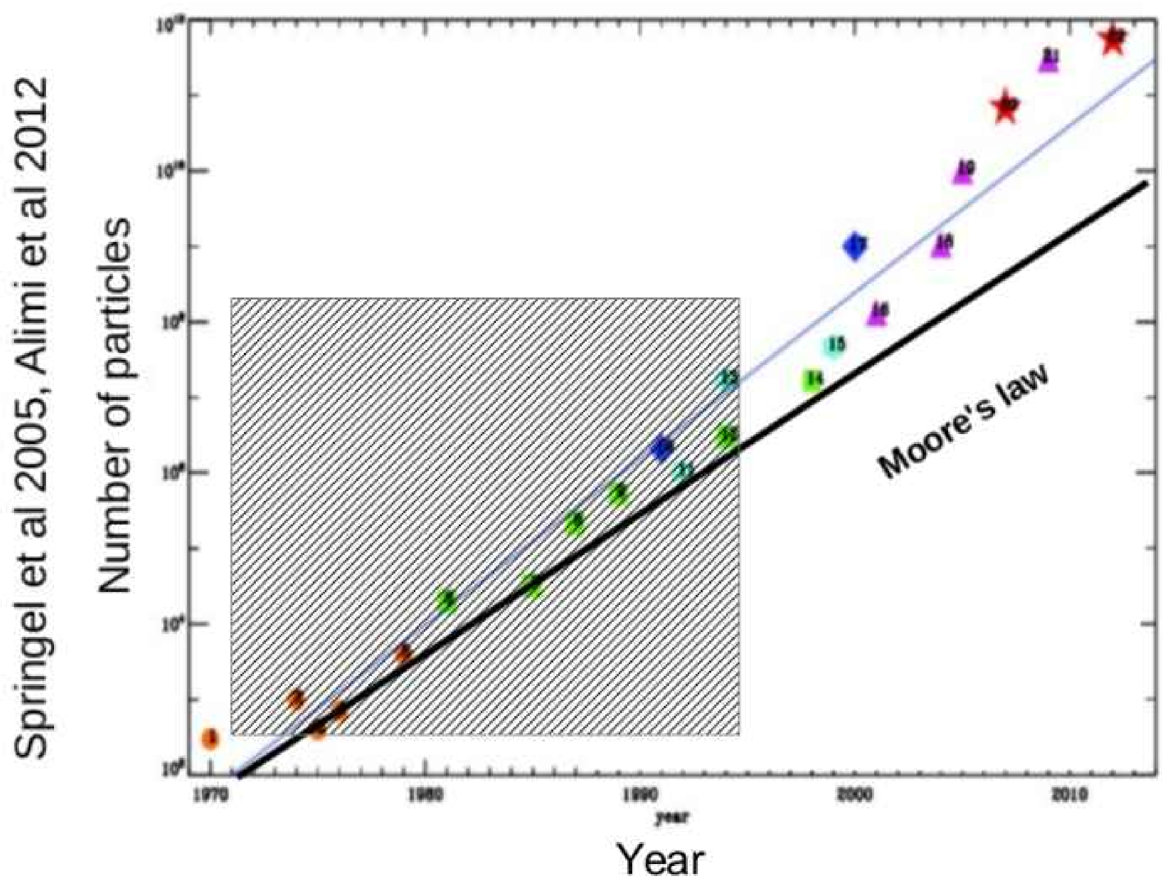
10 million CPUs
1.3 Pb of RAM
20 Pb of Disk
100 petaFOPS

273 million US dollars

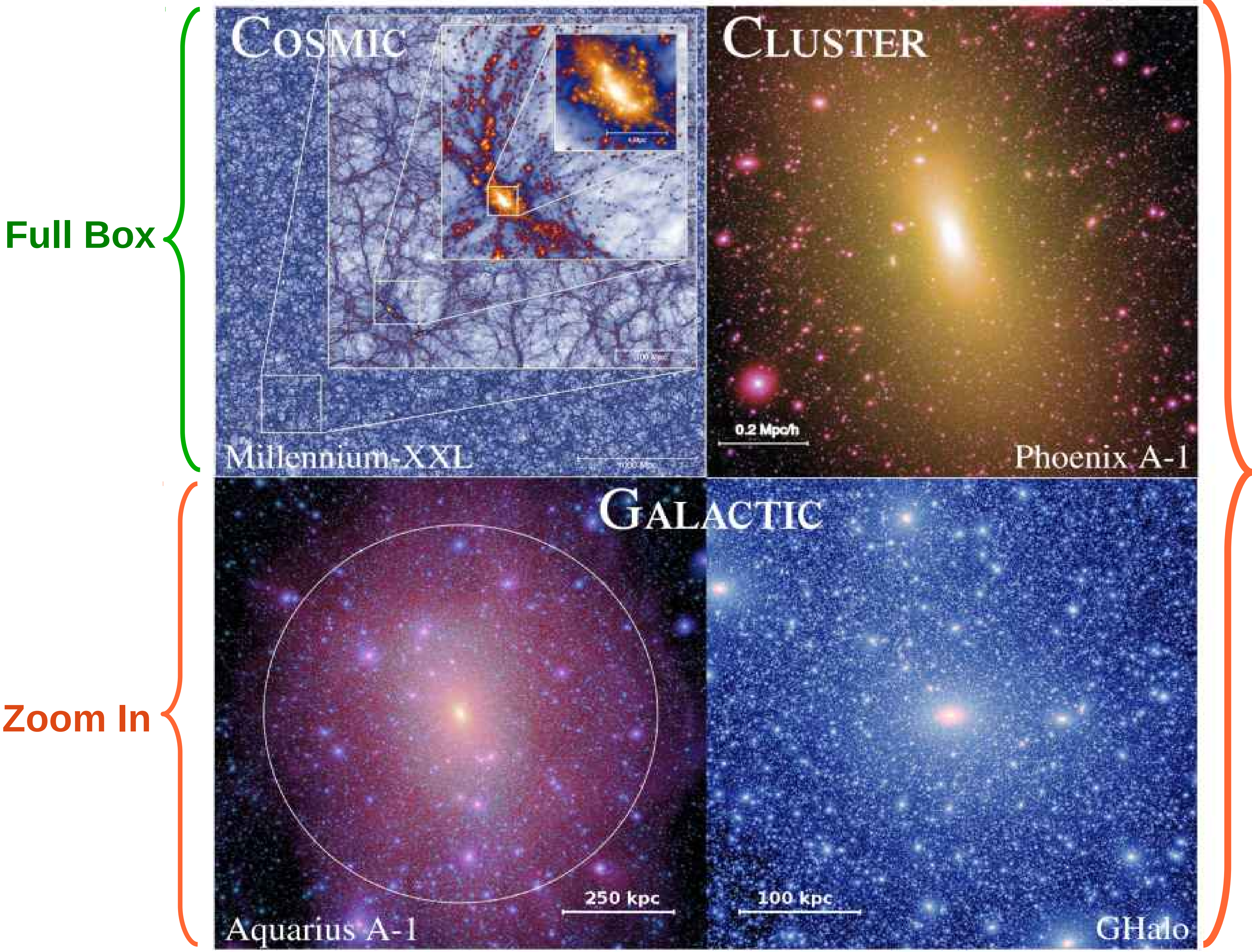
Exponential Growth of Computing Power

Exascale supercomputing and 10-trillion particle Simulations are expected to be reached by 2020

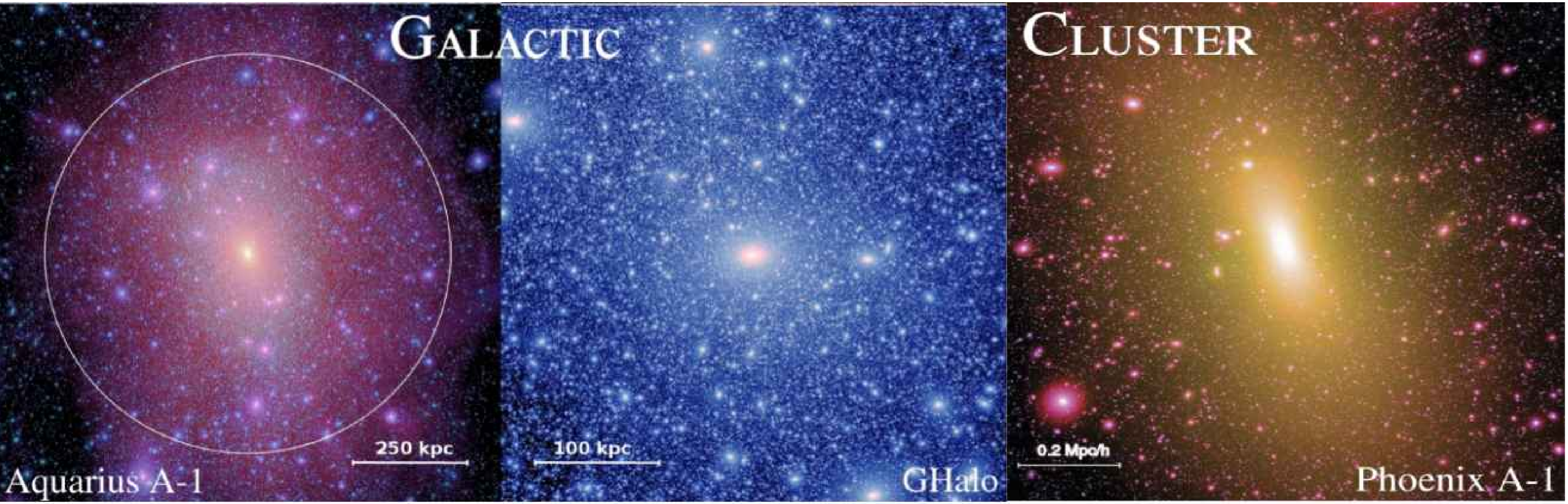




273 million US dollars



Full Box
Zoom In
Cosmic
Cluster
Galactic
Millennium-XXL
0.2 Mpc/h
Phoenix A-1
250 kpc
Aquarius A-1
100 kpc
GHalo

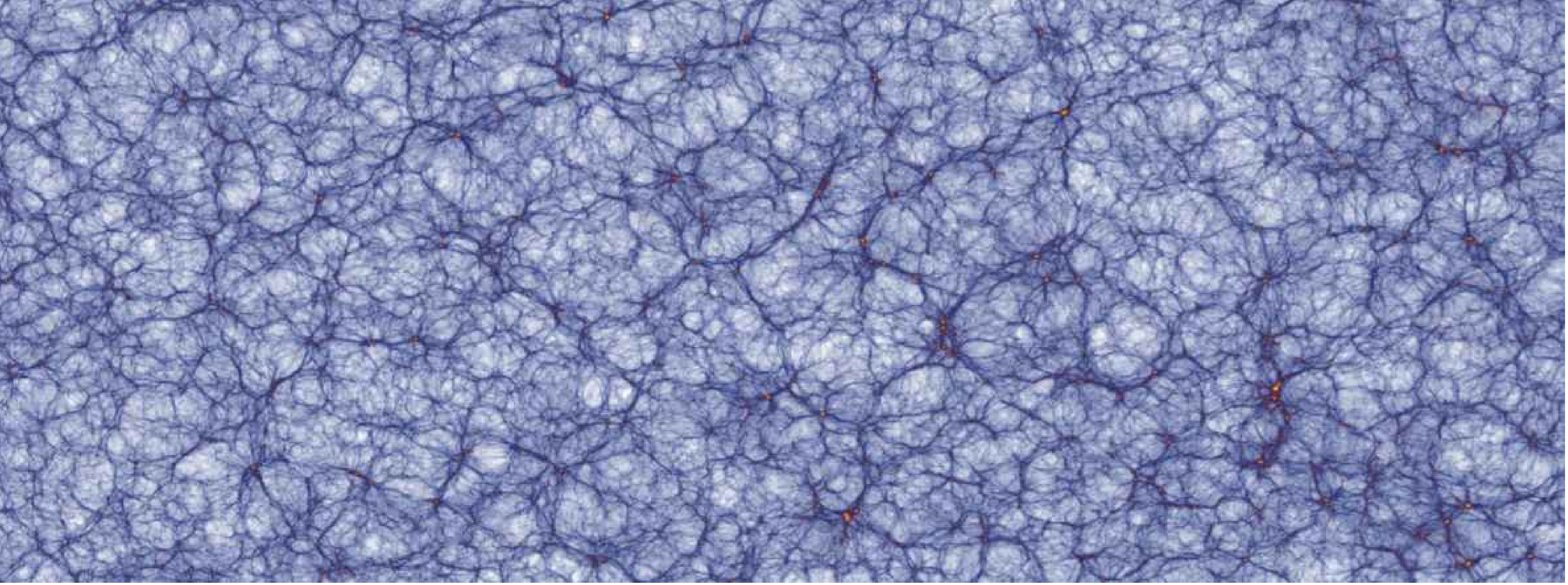


Springle+ 2008 Stadel+ 2009 Gao+ 2012

Zoom-In N-body simulations aim to predict:

- → Halo density and velocity profiles
- → Substructure mass function
- → Substructure spatial distribution

Direct Detection
Indirect Detection
Astrophysical Probes



Large-scale N-body simulations aim to predict:

→ The nonlinear state of mass

→ The velocity field

→ Abundance and properties of collapsed DM structures

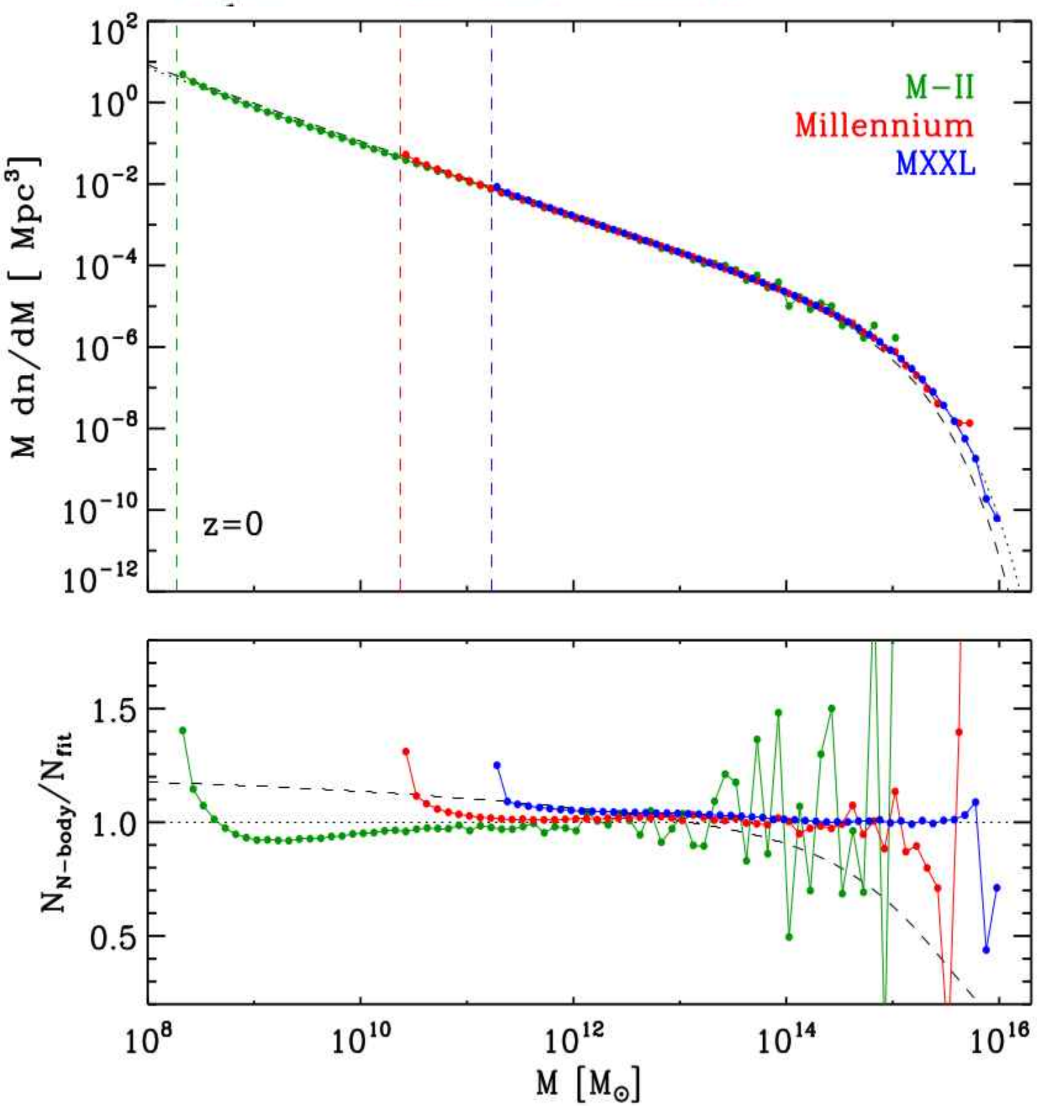
→ The places of galaxy formation

BAO & Galaxy Clustering
Abundance of Clsters
Weak Gravitational Lensing
Redshift-Space Distortions

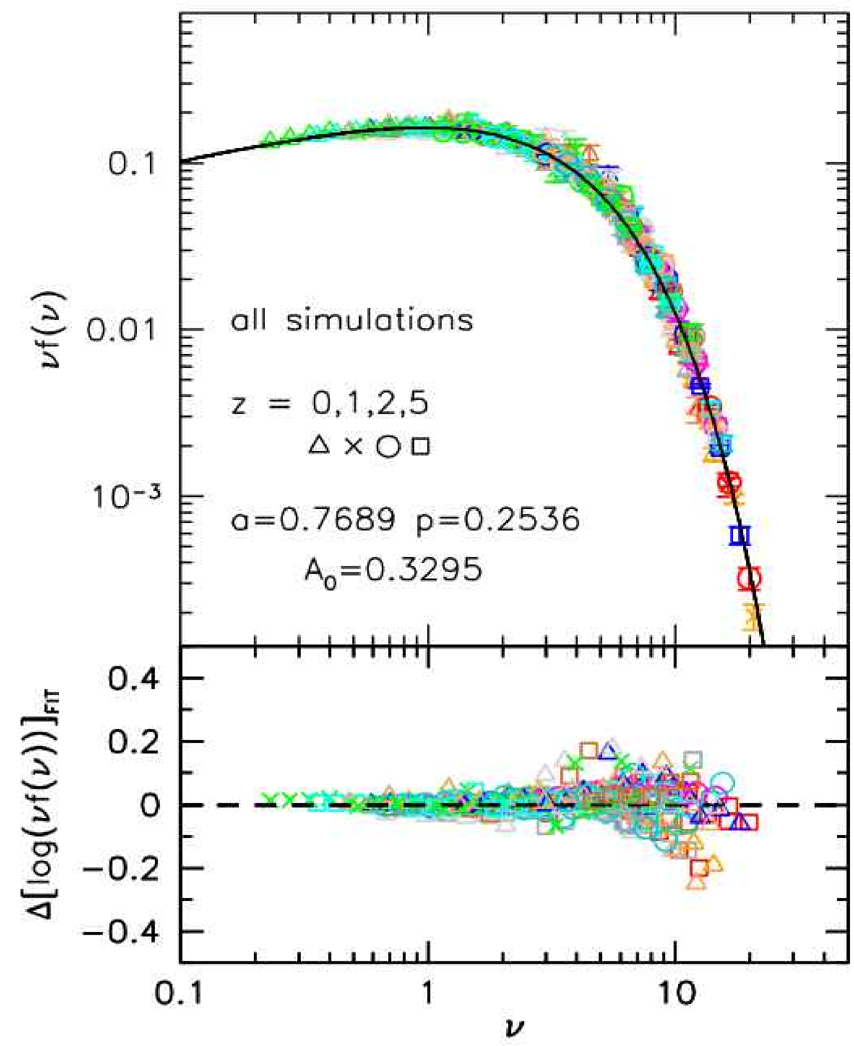
The abundance of CDM collapsed structures

Simulations resolve the mass range relevant for galaxy formation
If written in the adequate variables, the abundance is universal

$dn/dM \sim M^{-1.9}$ (at small masses)

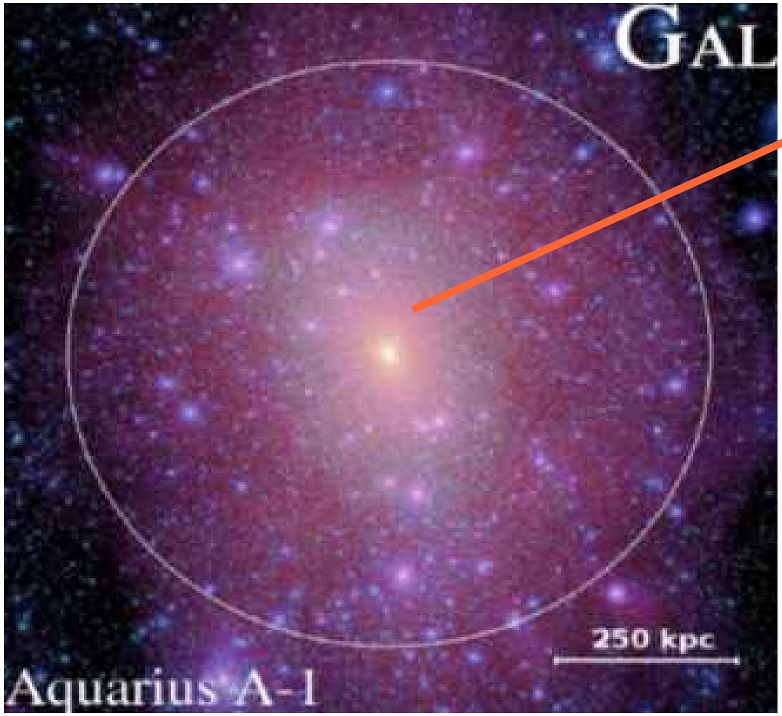


Angulo et al 2012



Despali et al 2015

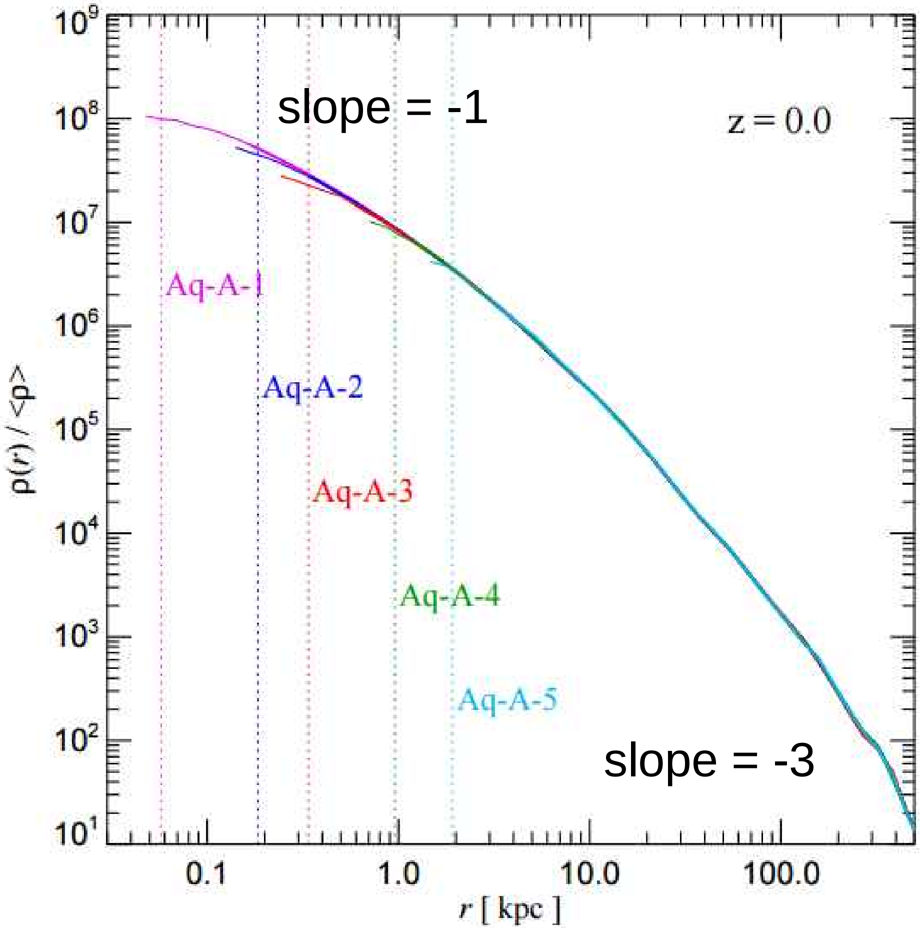
The inner structure of Dark Matter halos



Springel et al 2008

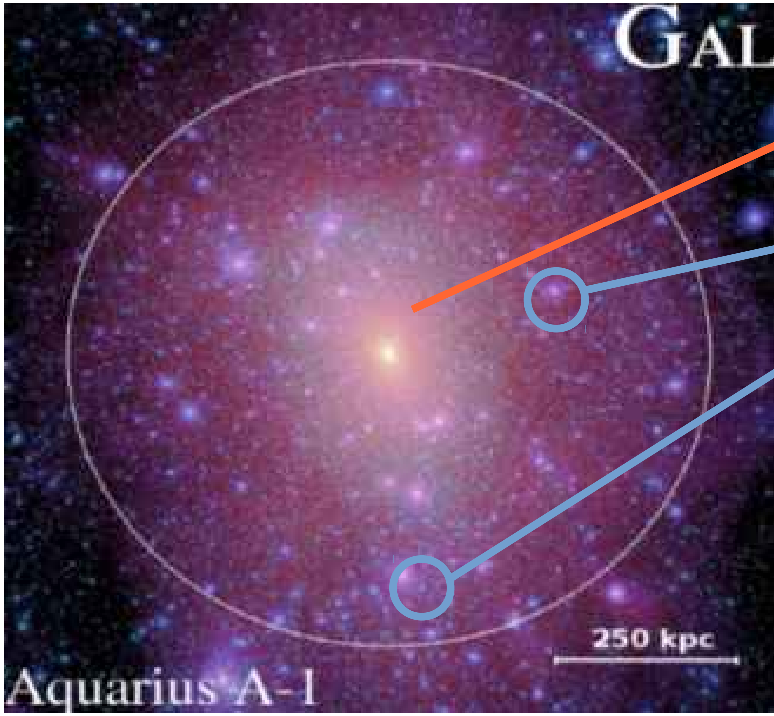
Smooth distribution

Density profile is described by NFW/Einasto functional form, independent of mass



Springel et al 2008

The inner structure of Dark Matter halos

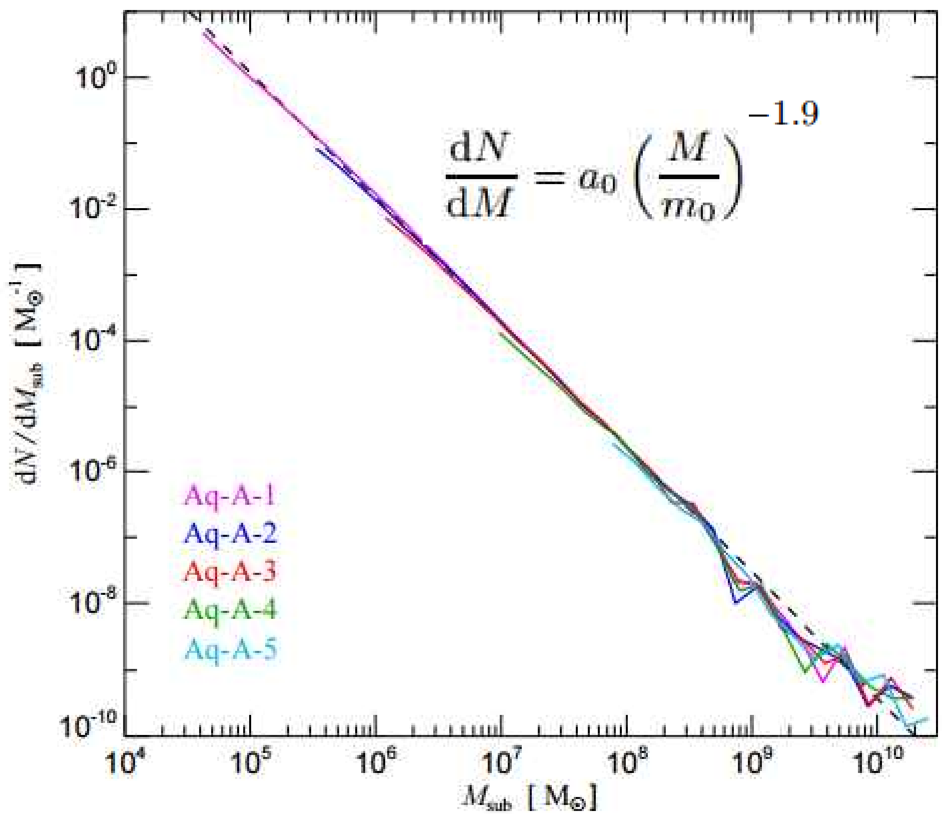


Springel et al 2008

Smooth distribution

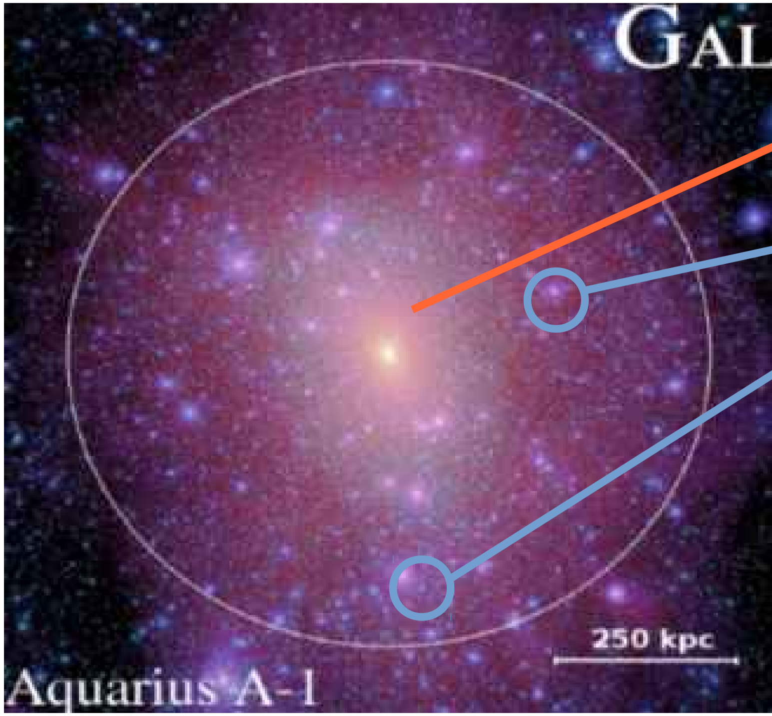
Hierarchy of substructures

→ **Abundance**



Springel et al 2008

The inner structure of Dark Matter halos

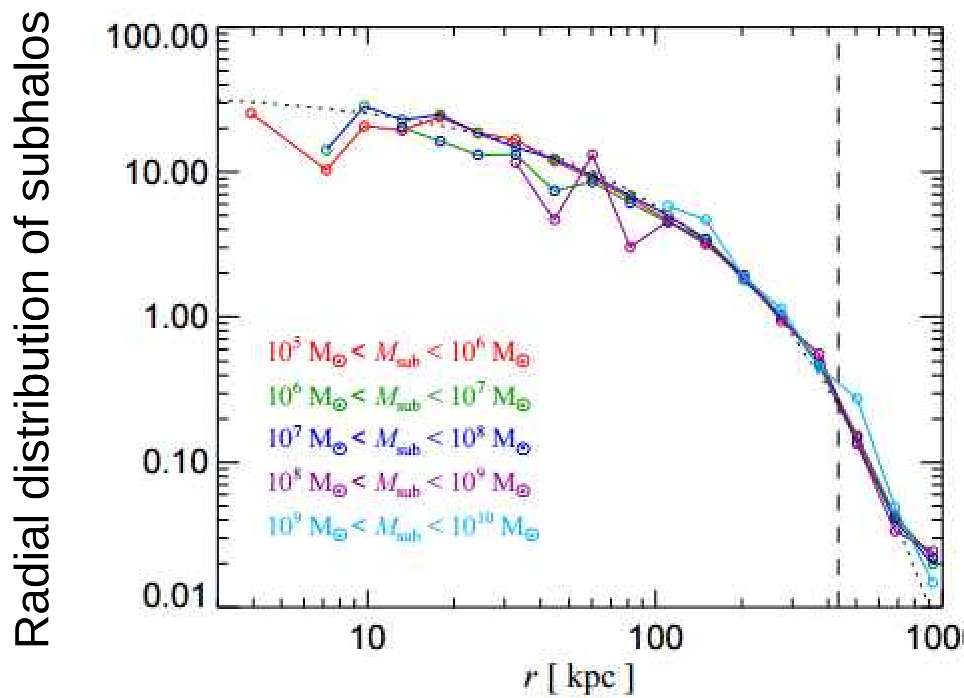


Springel et al 2008

Smooth distribution

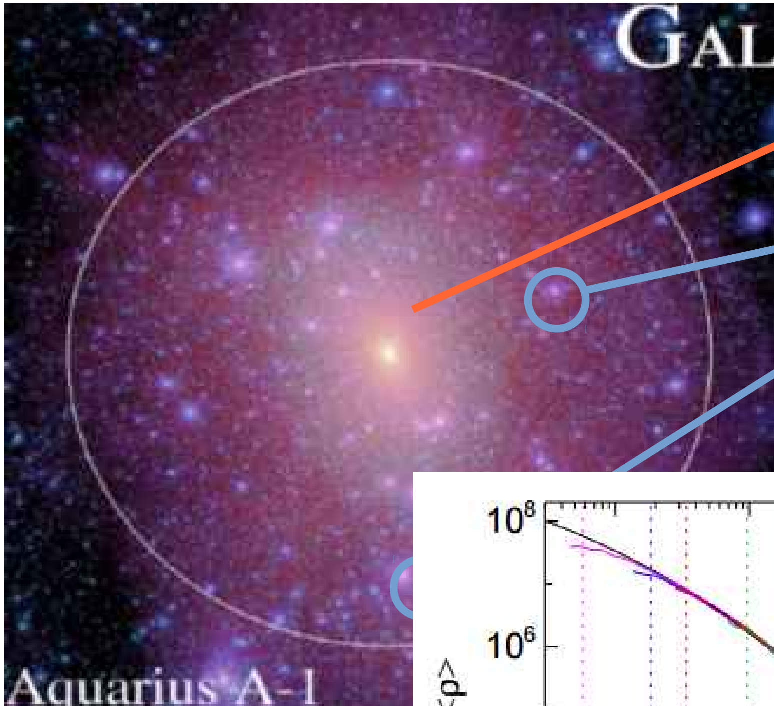
Hierarchy of substructures

→ **Abundance**
→ **Radial distribution**



Springel et al 2008

The inner structure of Dark Matter halos

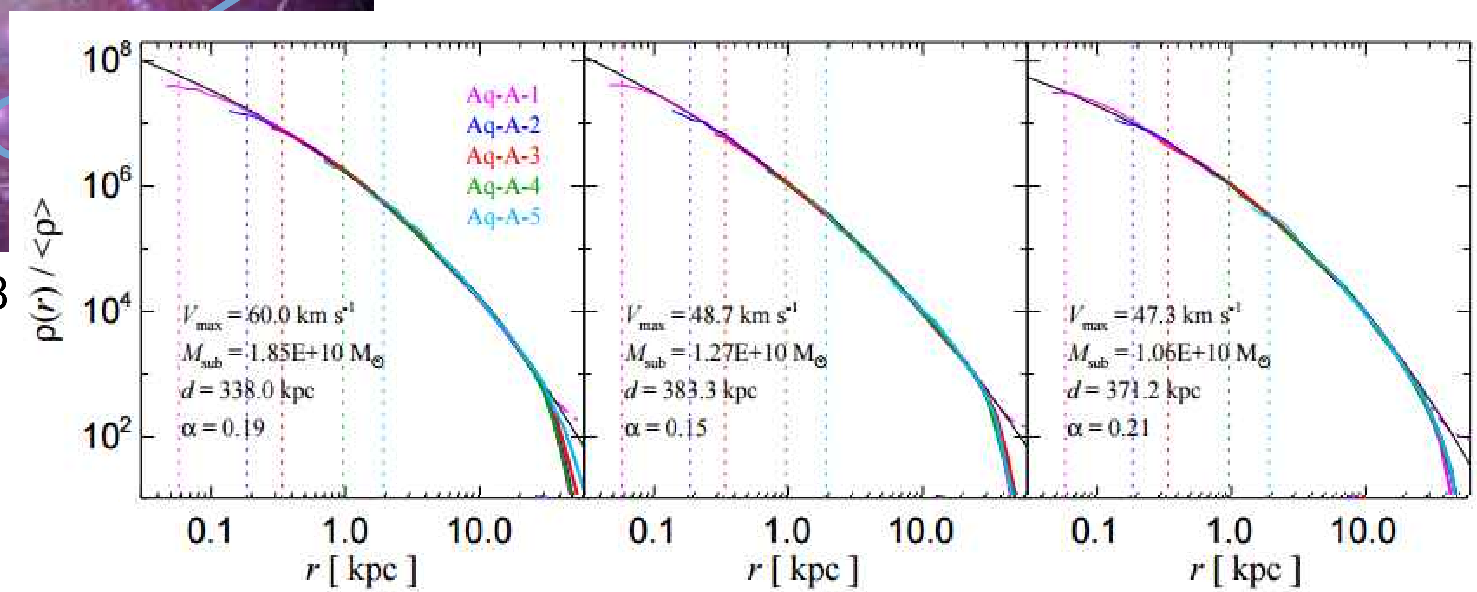


Springel et al 2008

Smooth distribution

Hierarchy of substructures

→ **Abundance**
→ **Radial distribution**
→ **Density profile**



→ Background

→ Methods

→ Current State of the Art

→ The next decade

→ Open questions & challenges

Simulating departures from LCDM

Many flavours of cosmological simulations dropping one or more of the assumptions of the simplest case

Cold	→ Warm
Collisionless	→ Self-Interacting DM
DM only	→ DM plus neutrinos
Gaussian IC	→ Primordial NG
GR	→ Modified gravity simulations
Classical particles	→ axion/wave/fussy DM

Simulating departures from LCDM

Many flavours of cosmological simulations dropping one or more of the assumptions of the simplest case

Cold

Collisionless

DM only

Gaussian IC

GR

Classical particles

Self-interacting Dark Matter

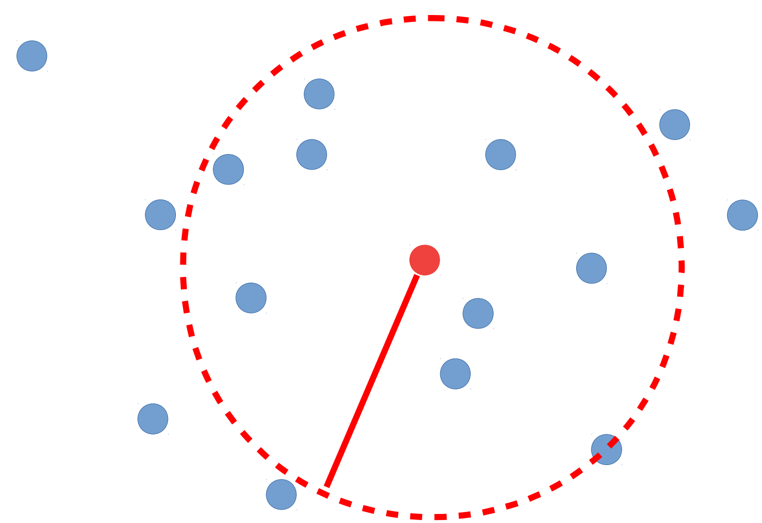
A cross-section for isotropic & elastic scattering among DM particles could lead to modifications to small-scale structure

$$\frac{\mathrm{d}f(\mathbf{x},\mathbf{u},t)}{\mathrm{d}t} = \Gamma_{in} - \Gamma_{out}$$

$$\Gamma_{out} = f(\mathbf{x},\mathbf{u}) \int \mathrm{d}\mathbf{u}\, f_v(\mathbf{x},\mathbf{u},t)\rho(\mathbf{x})\, \frac{\sigma_\chi}{m}\, |\mathbf{u}-\mathbf{u}_i|$$

Implementation

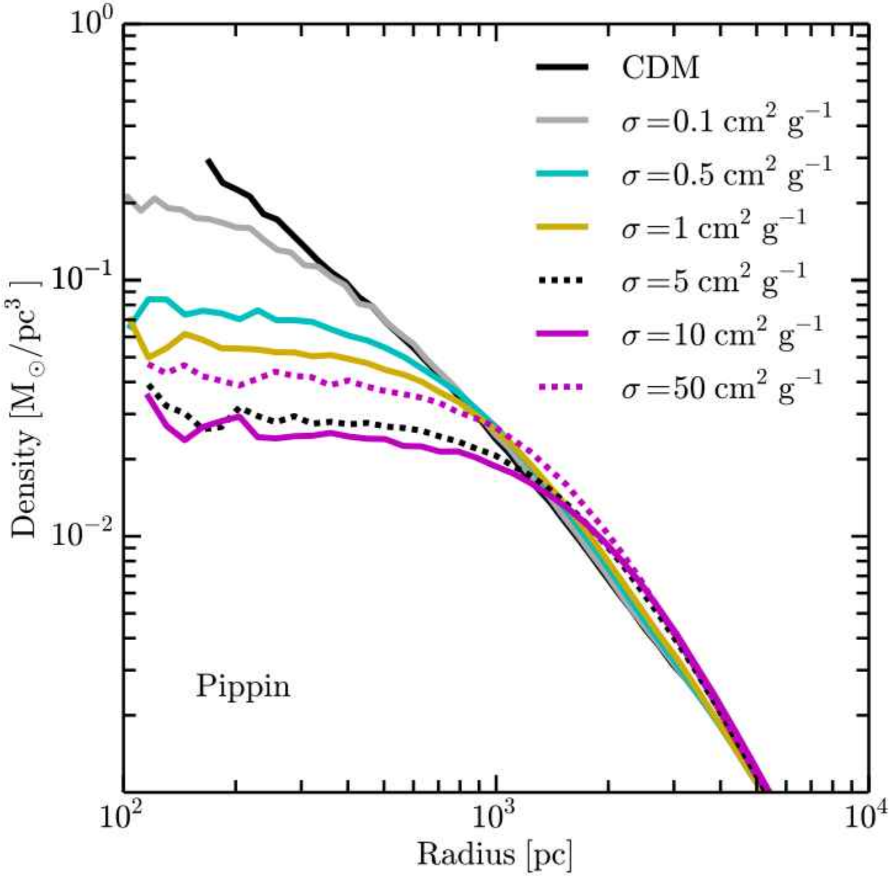
$$\Gamma^j_{out} = \sum_i^N \frac{3}{4\pi\epsilon^3}\sigma m_p |\mathbf{v}_j - \mathbf{v}_i|$$

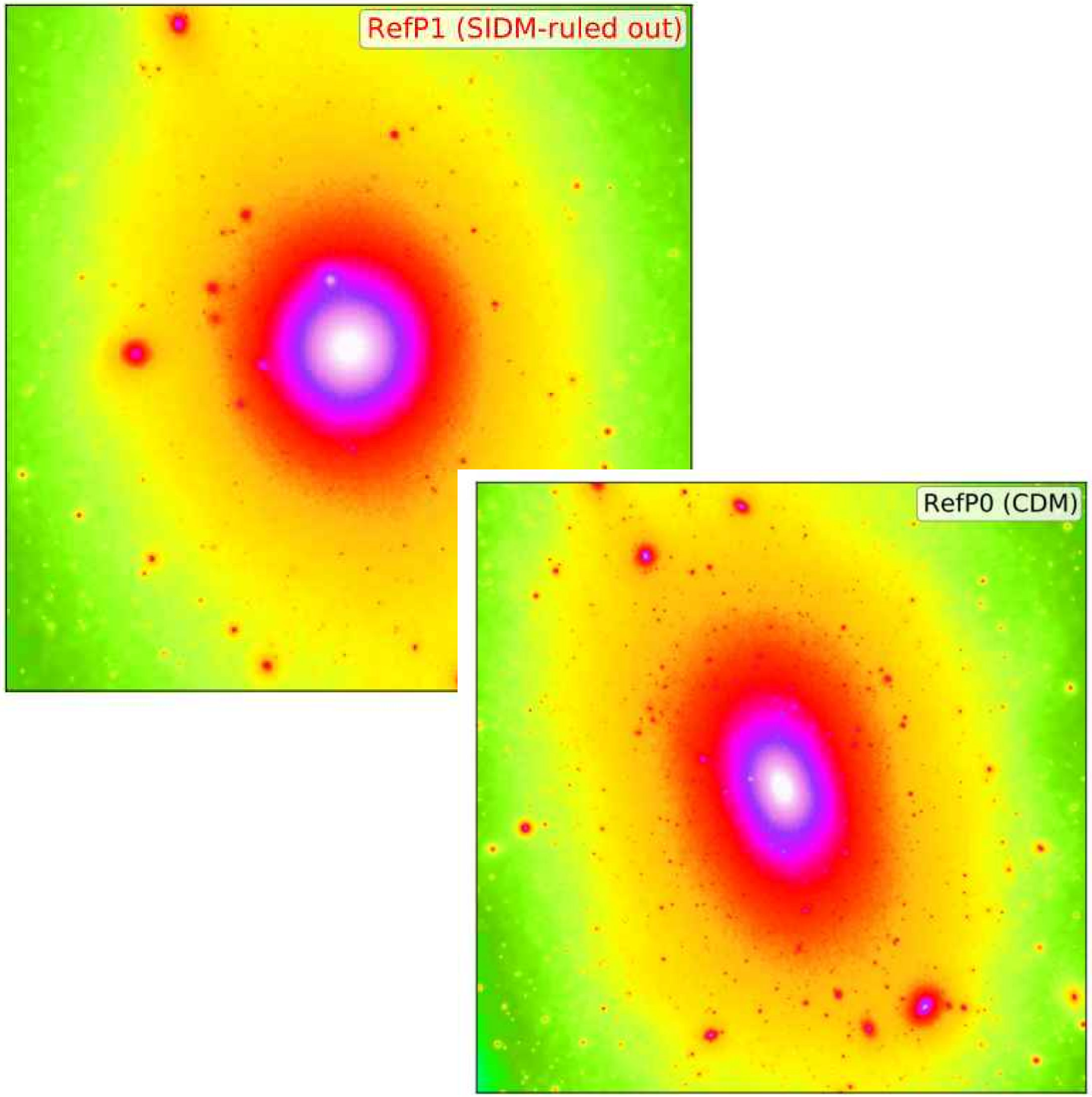


Self-interacting Dark Matter

A large cross-section reduces the central density in dark matter halos and make them rounder

Elbert et al (2015)

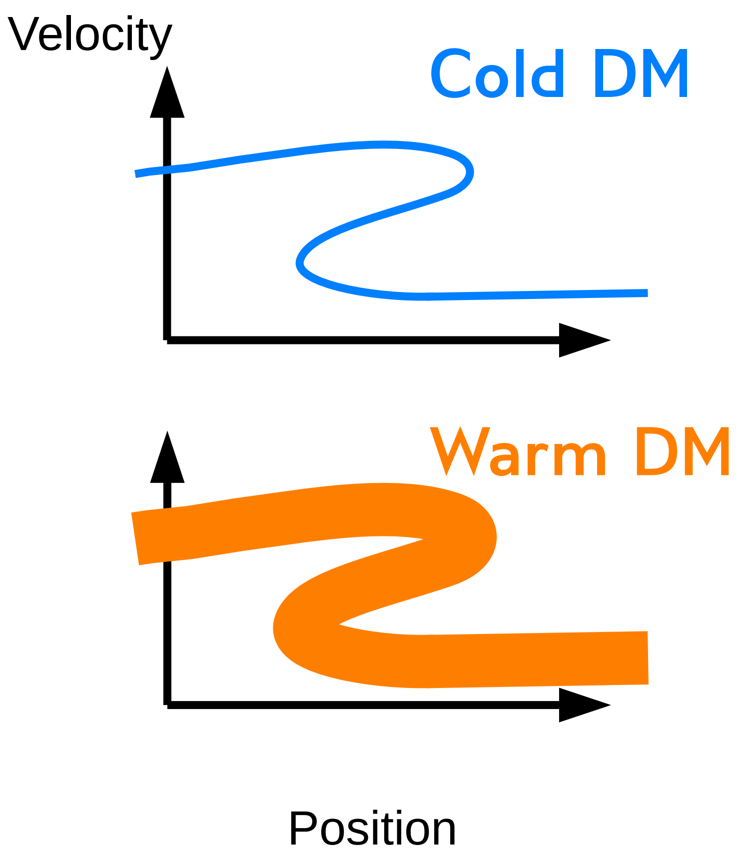


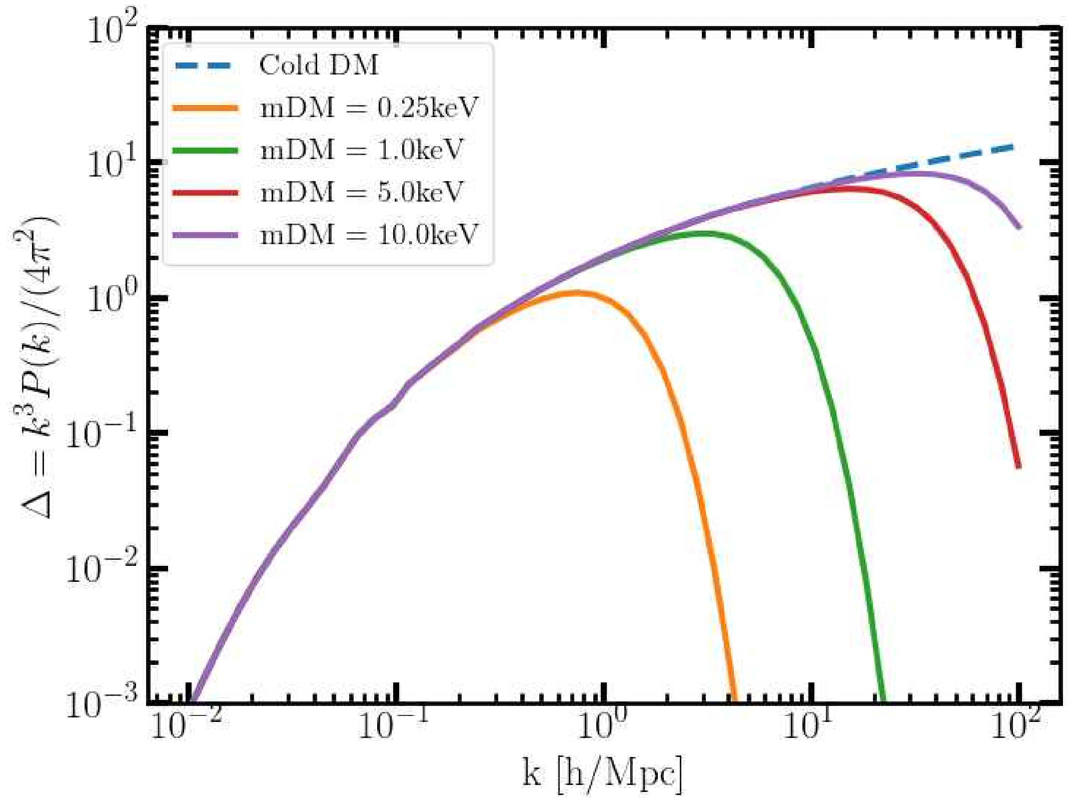


Vogelsberger et al (2012)

Warm Dark Matter

Free streaming of particles out of overdensities erases primordial fluctuations on small scales





Implementation: Since the extent in velocity space is quite small, typical N-body simulations assume WDM as Cold with a modified initial power spectrum

Warm Dark Matter

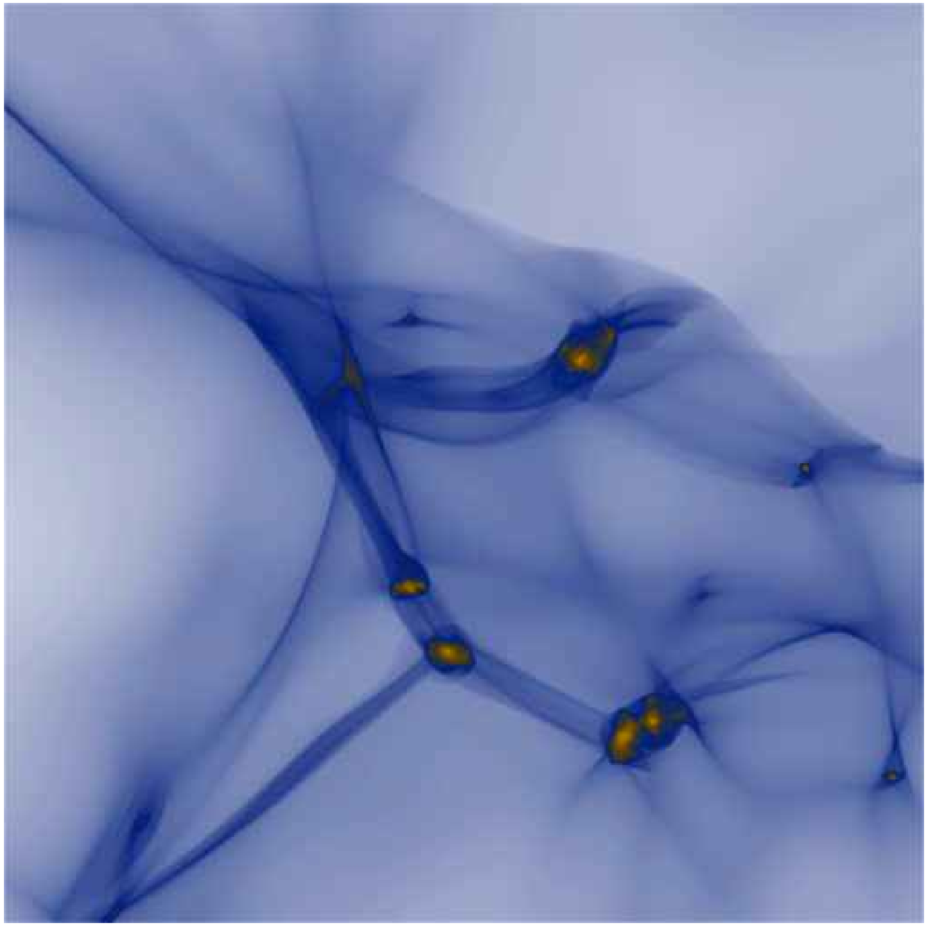
Free streaming of particles out of overdensities erases primordial fluctuations on small scales

No bottom-up formation

Genuine 1D and 2D structures

Halos have no progenitors beyond some time

Very rapid mass growth during formation

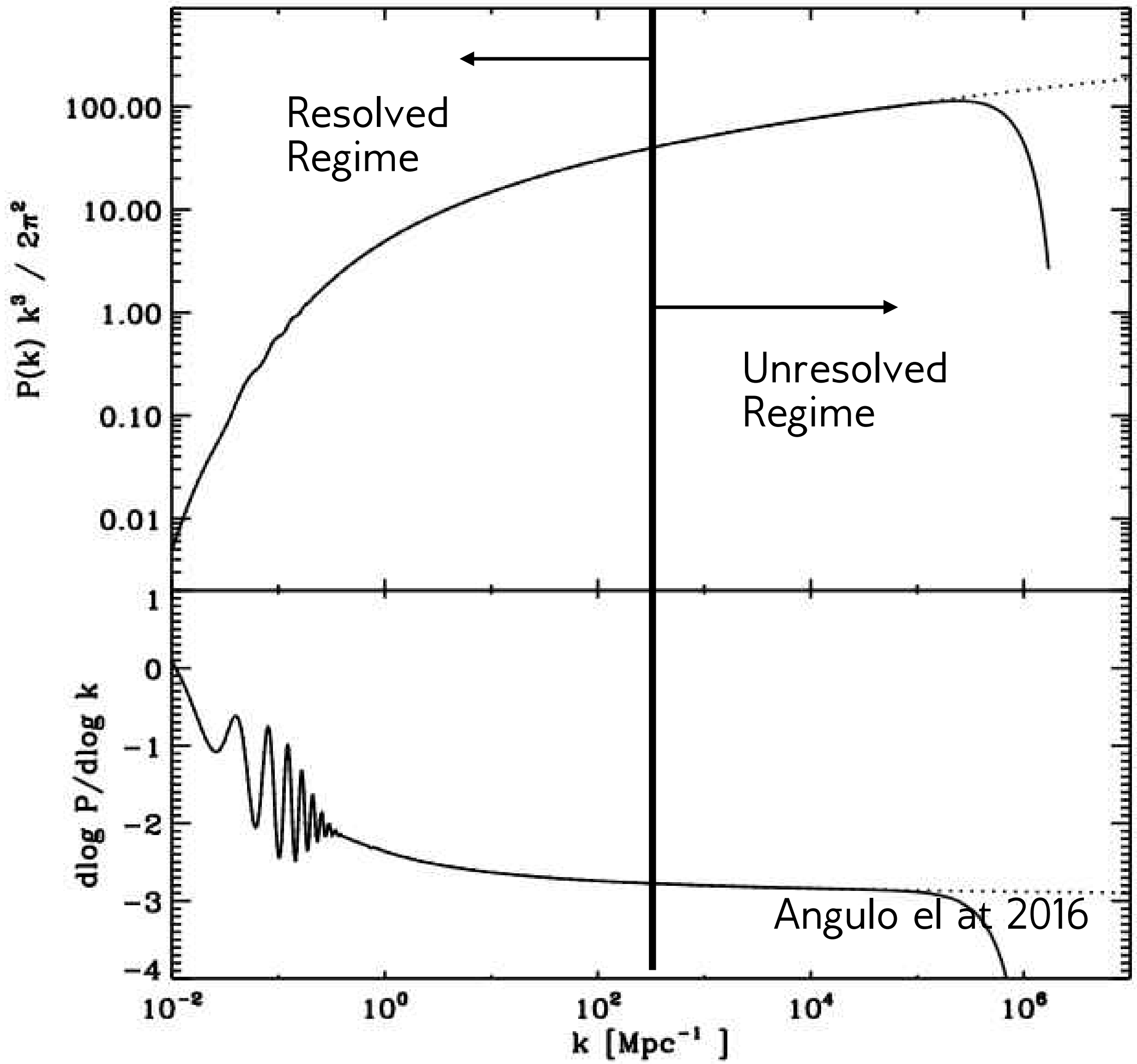


Angulo et al (2012)

CDM at the free-streaming scale

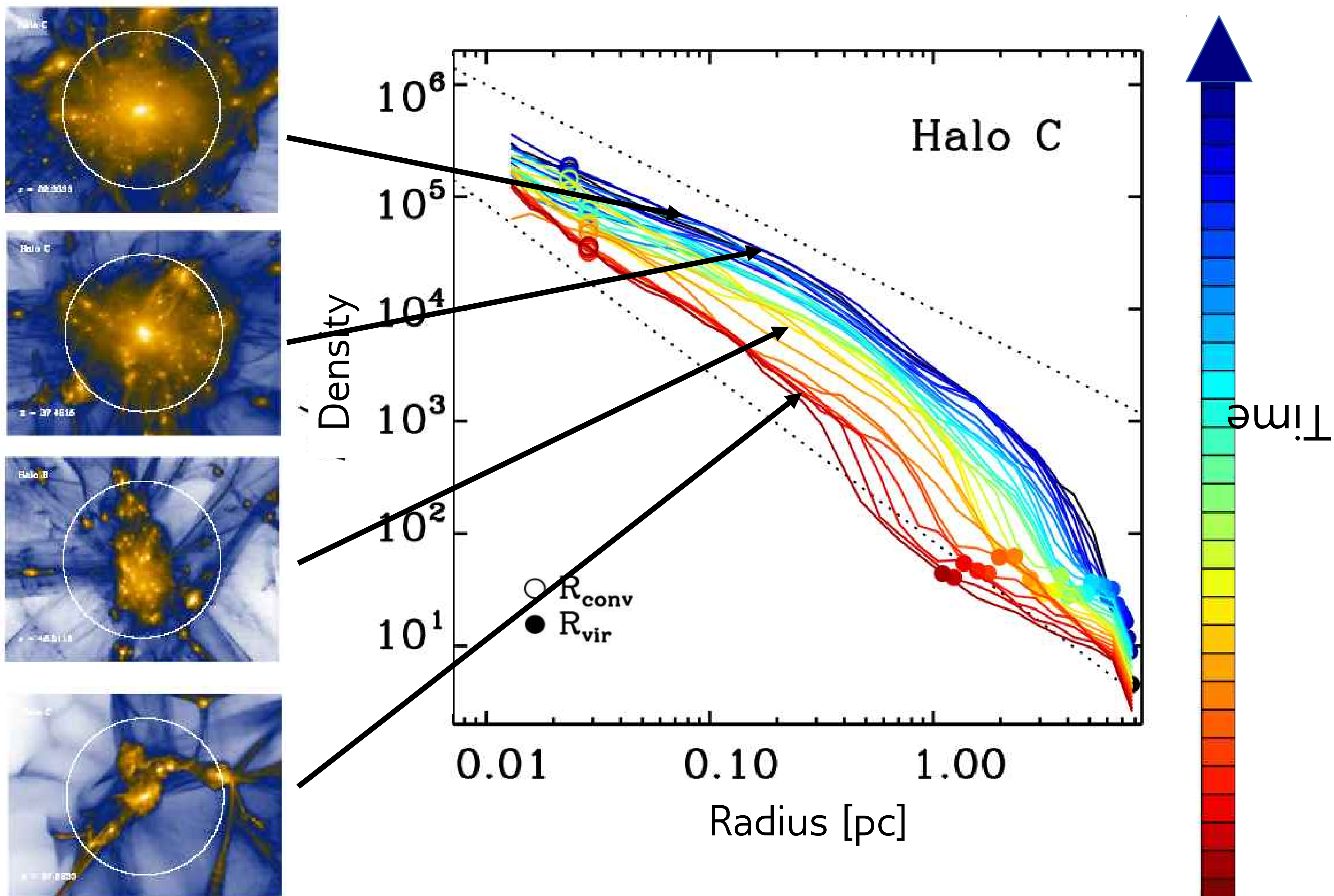
For a 100GeV Neutralino, halos down to 1 earth mass form

Angulo & White 2010




z = 399.00

Structure formation at the free streaming mass

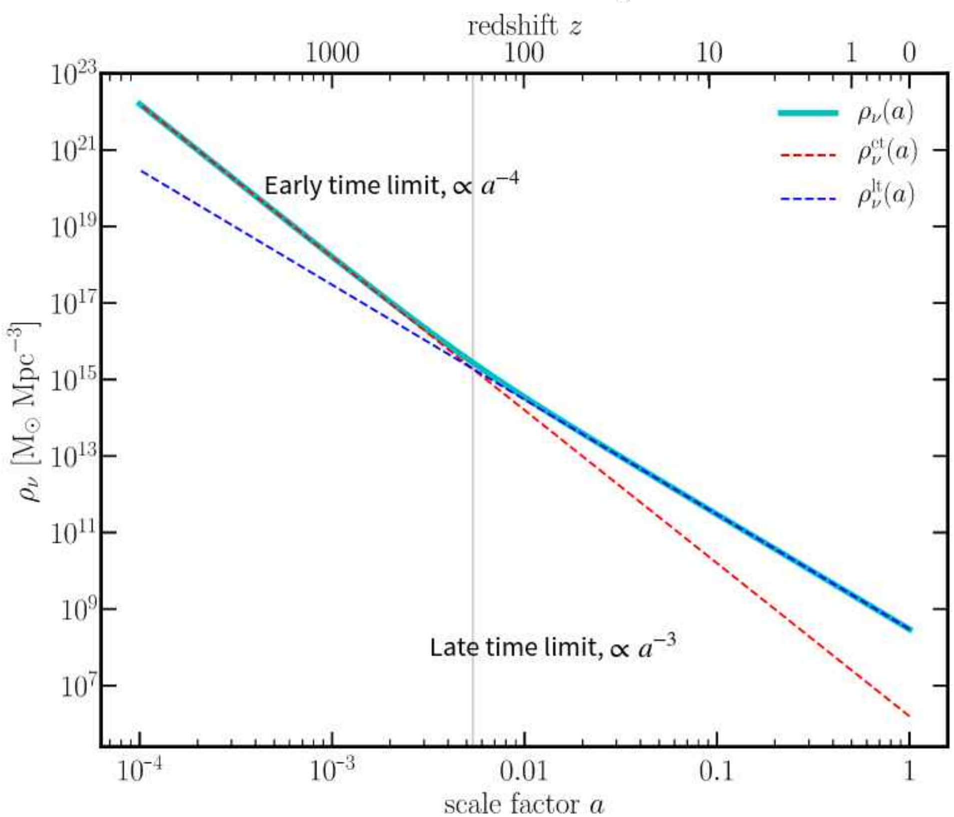


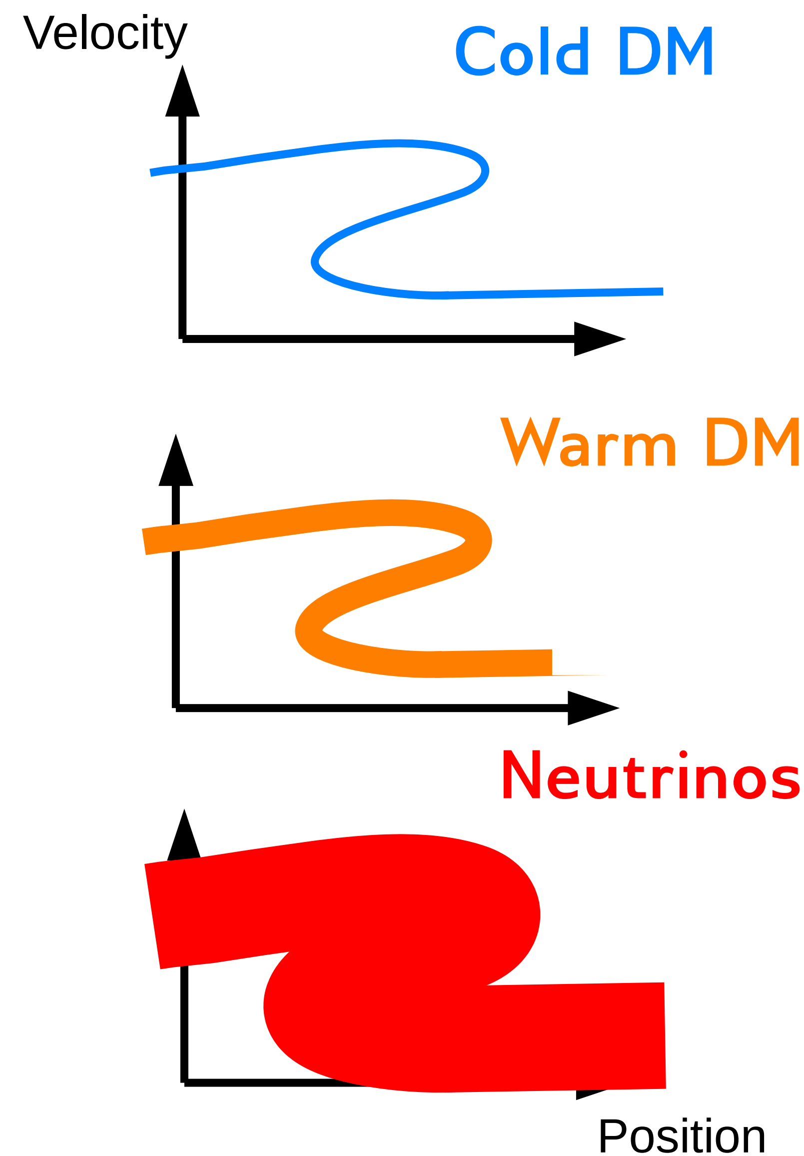
Massive Neutrinos

Current constraints indicate masses between 0.05 and 0.15eV
Even with hints of detection

$$f_\nu(\mathbf{u}, t) = (\exp(\mathbf{u}c/k_b T_\nu) + 1)^{-1}$$

$$\Omega_\nu h^2 = \frac{1}{93.14eV} \sum_i m_{\nu,i}$$



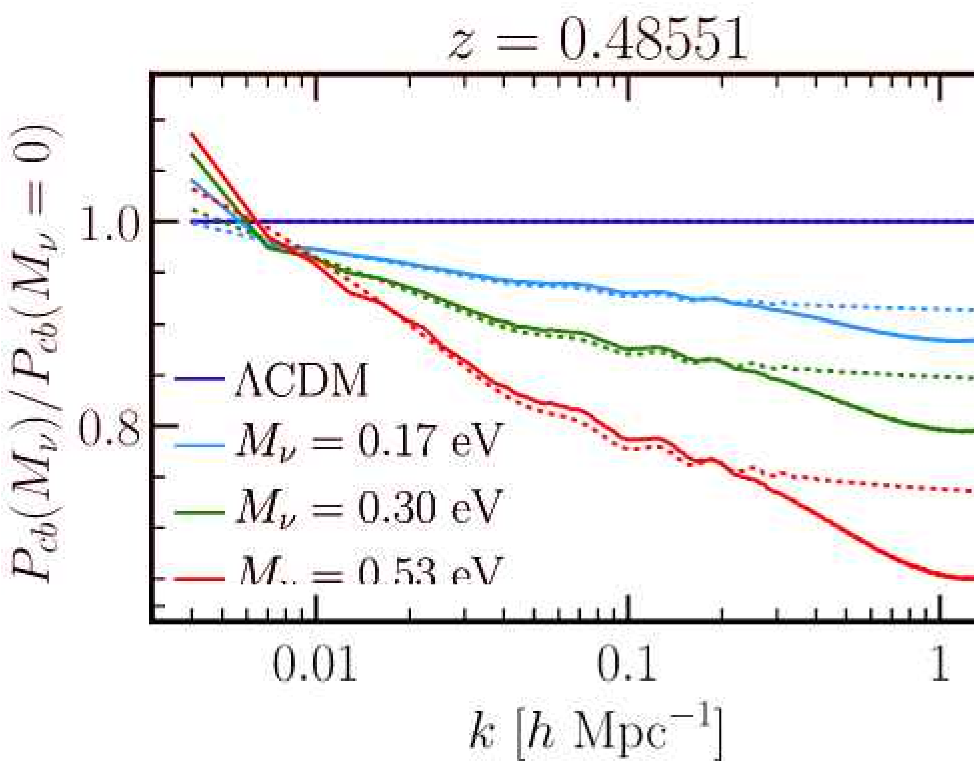


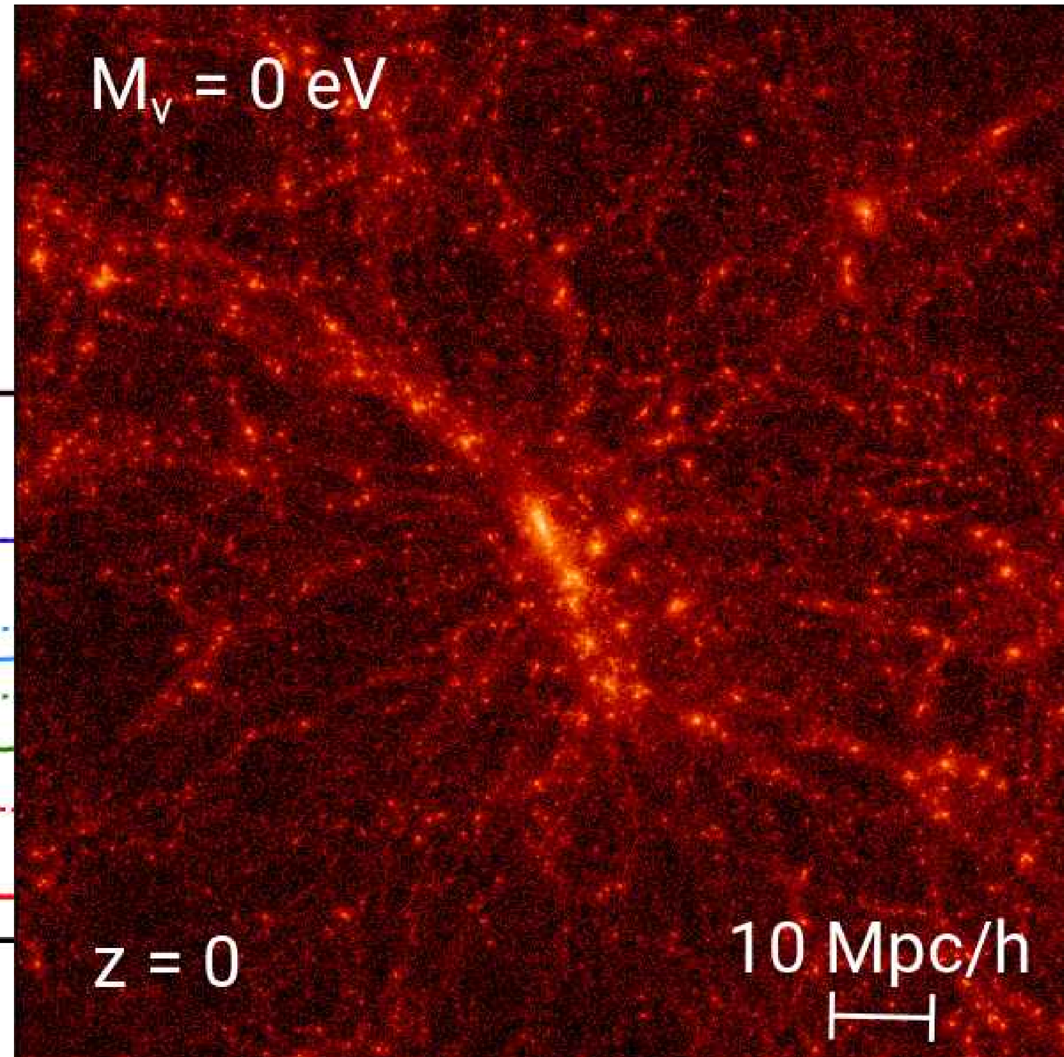
Massive Neutrinos

Neutrinos suppress structure growth on small scales

Implementation:

a) Solve linearised VP on a grid
b) MonteCarlo Sample f(v)
c) hybrid methods





Credit: Matteo Zennaro

→ Background

→ Methods

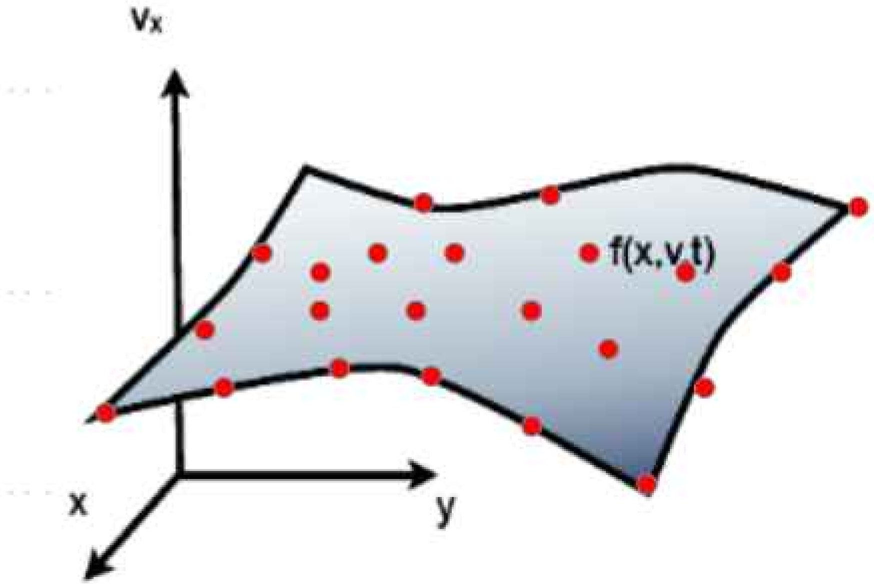
→ Current State of the Art

→ **The next decade:**
i) New VP solvers &
ii) Cosmological Parameters

→ Open questions & challenges

Standard approach to solving the VP equation:

Montecarlo Sampling and coarse graining the CDM distribution function

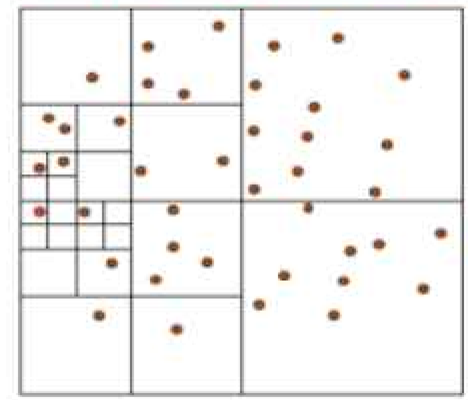


$$\frac{\mathrm{d}^2\mathbf{x}_i}{\mathrm{d}^2 t} = \nabla_i \Phi(\mathbf{x}_i),$$

$$\Phi(\mathbf{x}) = -G \sum_i \frac{m_i}{[(\mathbf{x}_i - \mathbf{x})^2 + \varepsilon^2]}$$

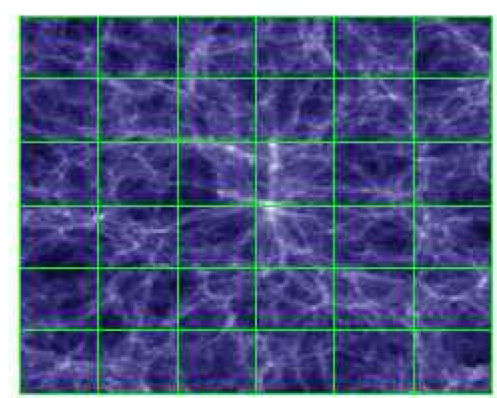
Tree Algorithms

Multipole decomposition



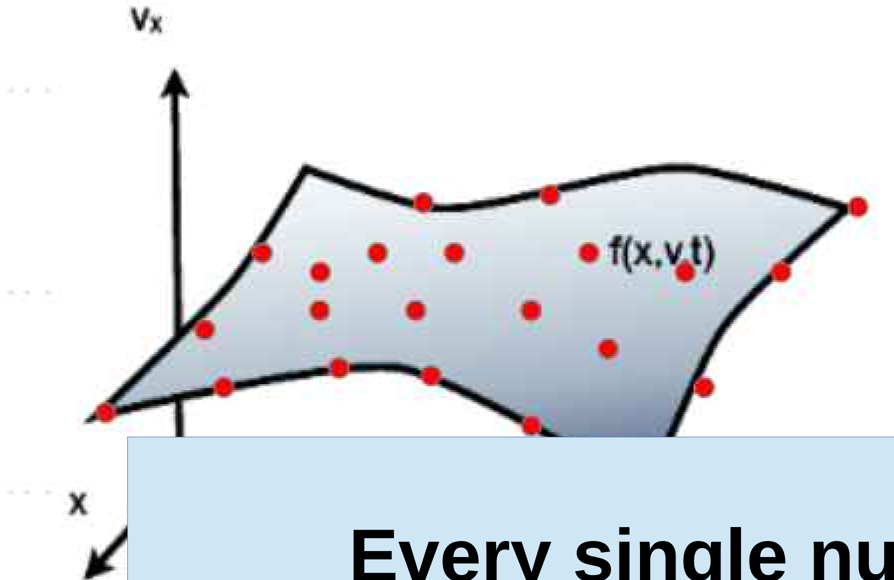
Particle-Mesh

Poisson equation



Standard approach to solving the VP equation:

Montecarlo Sampling and coarse graining the CDM distribution function

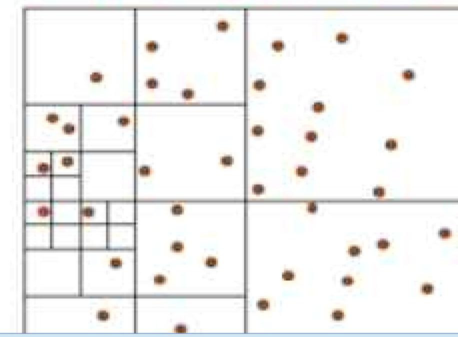


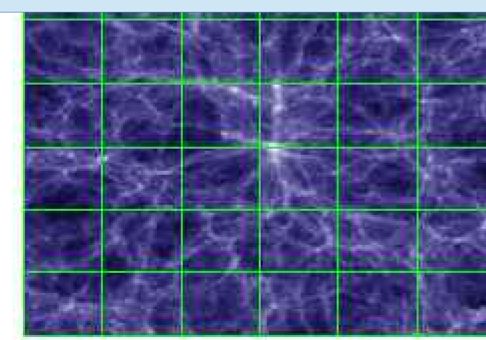
$$\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}^2t}$$

$$\Phi(\mathbf{x}) = -G\sum_i \frac{m_i}{[(\mathbf{x}_i - \mathbf{x})^2 + \varepsilon^2]}$$

Tree Algorithms

Multipole decomposition





Every single numerical simulation out there (even SPH/AMR) relies on the same assumption

Two examples where the N-body fails:

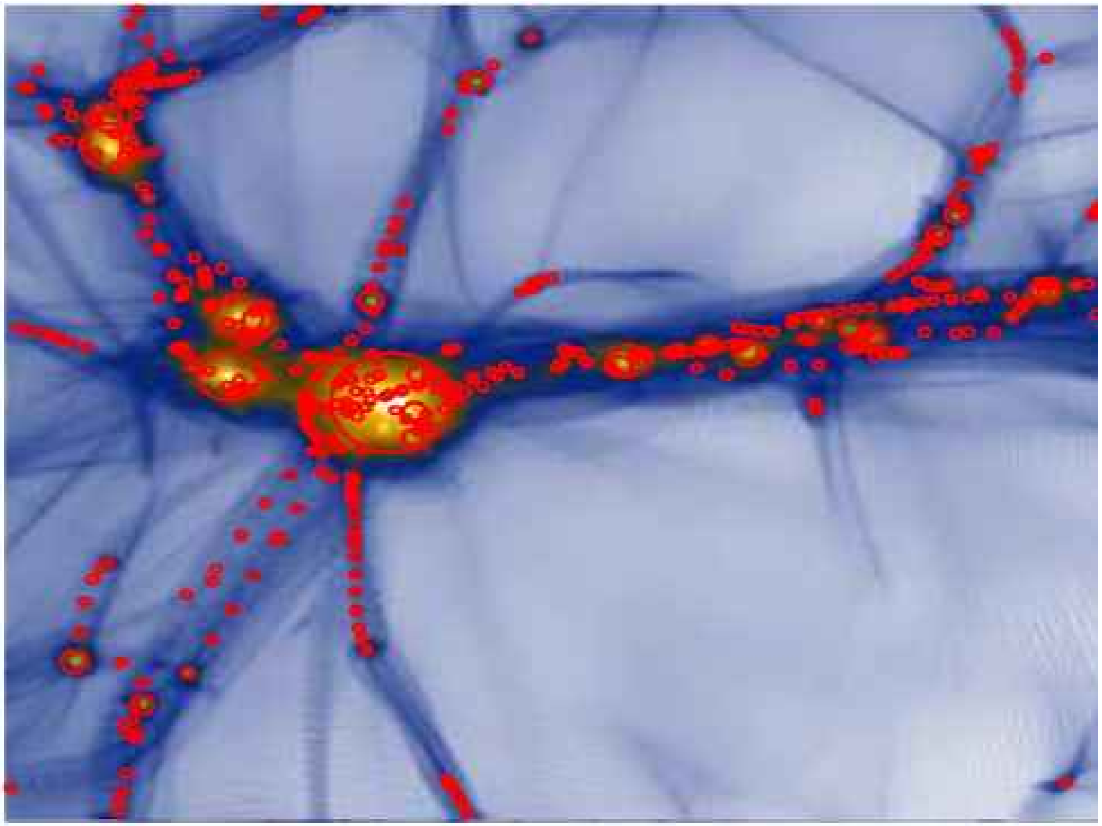
1) Two fluids with distinct primordial power spectra
2) Artificial fragmentation of filaments

$$\Phi(\mathbf{x}) = -G \sum_i \frac{m_i}{[(\mathbf{x}_i - \mathbf{x})^2 + \varepsilon^2]}$$

Two competeting requirements For setting epsilon

i) A *large* ε value to reduce noise.

ii) A *small* ε value to resolve structures



The evolution of the fine and coarse grained distribution functions are NOT equivalent.

1) Two fluids with distinct primordial power spectra
2) Artificial fragmentation of filaments

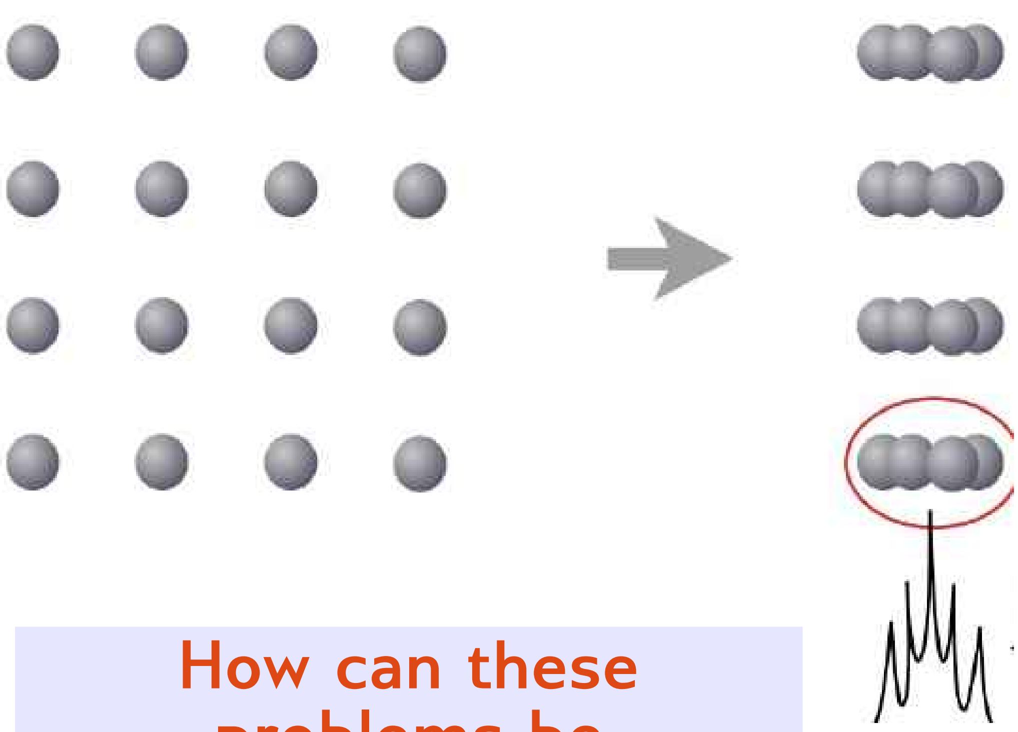
$$\Phi(\mathbf{x}) = -G \sum_i \frac{m_i}{[(\mathbf{x}_i - \mathbf{x})^2 + \varepsilon^2]}$$

Two competing requirements
For setting epsilon

i) A *large* ε value
 to reduce noise.

ii) A *small* ε value to
 resolve structures

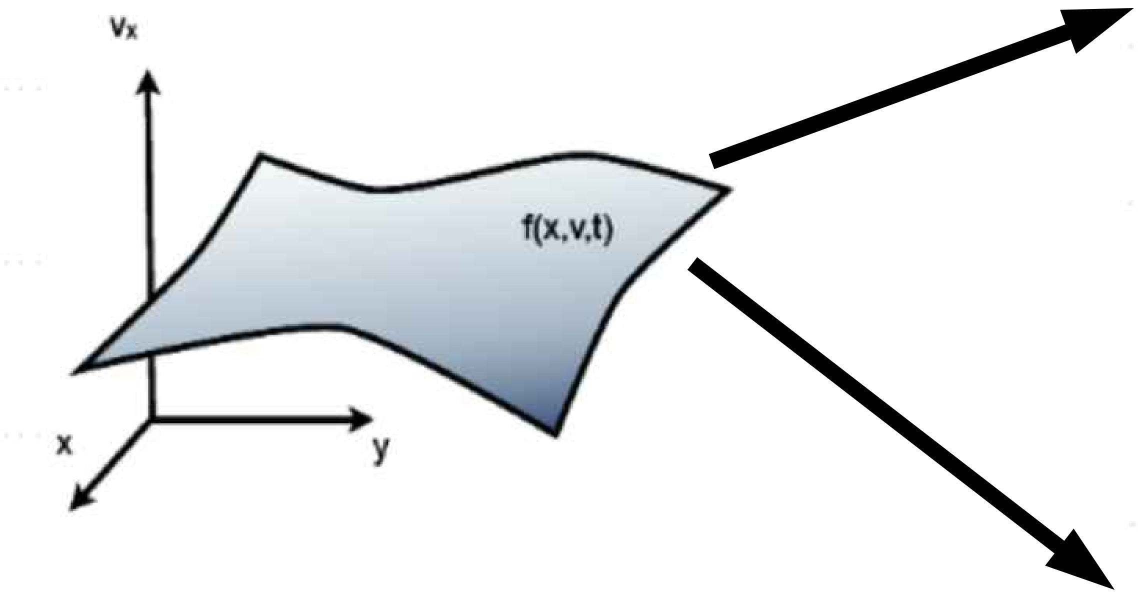
Anisotropic compression in triaxial collapse

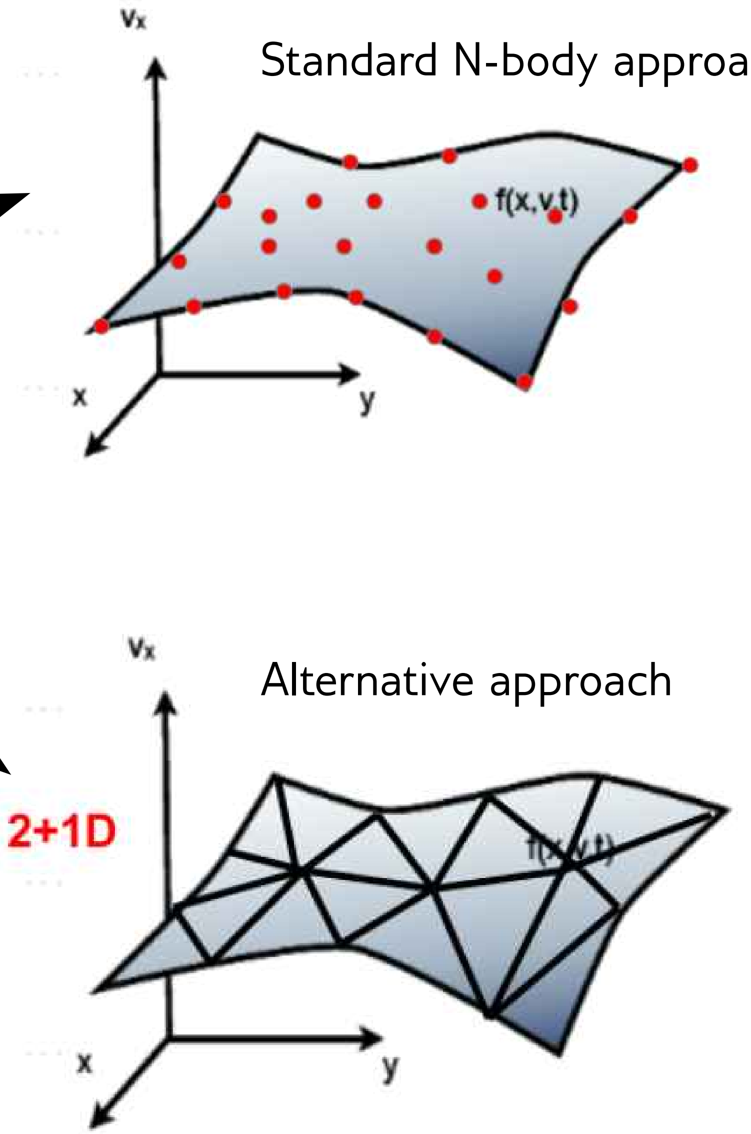


How can these problems be cured/tested?

Tessellation of the DM fluid with phase-space Lagrangian elements

(Abel+ 2012, Shandarin+ 2012, Kaehler+ 2013, Hahn+ 2013, Angulo+ 2013, Hahn & Angulo 2014)

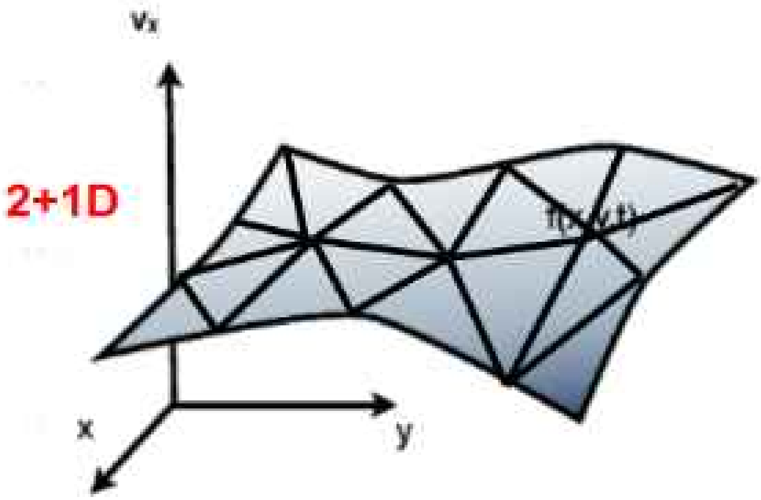




A tessellation of a finite number of mesh-generating points in Lagrangian space allows to continuously map the deformation of the dark matter sheet

Warm Dark Matter structure formation without noise (Angulo, Hahn, Abel 2013b)

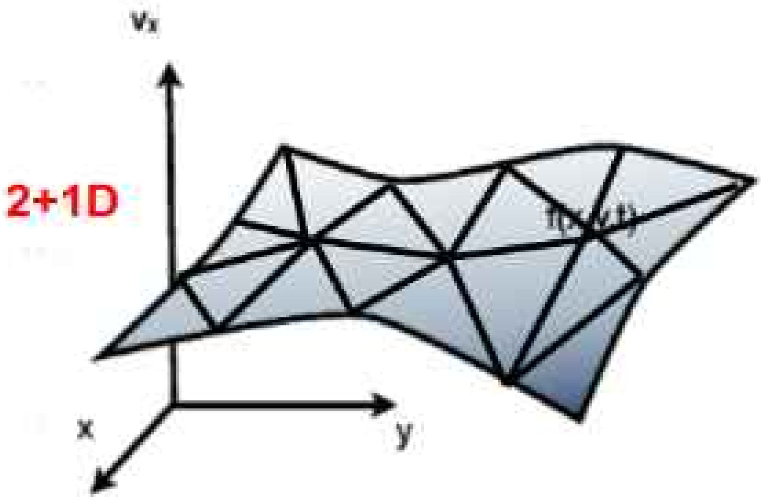
New sheet-based simulation code with reduced collisionality and noise



$$\nabla^2 \Phi = \frac{4\pi G}{a} \int f \mathrm{d}^3 v$$

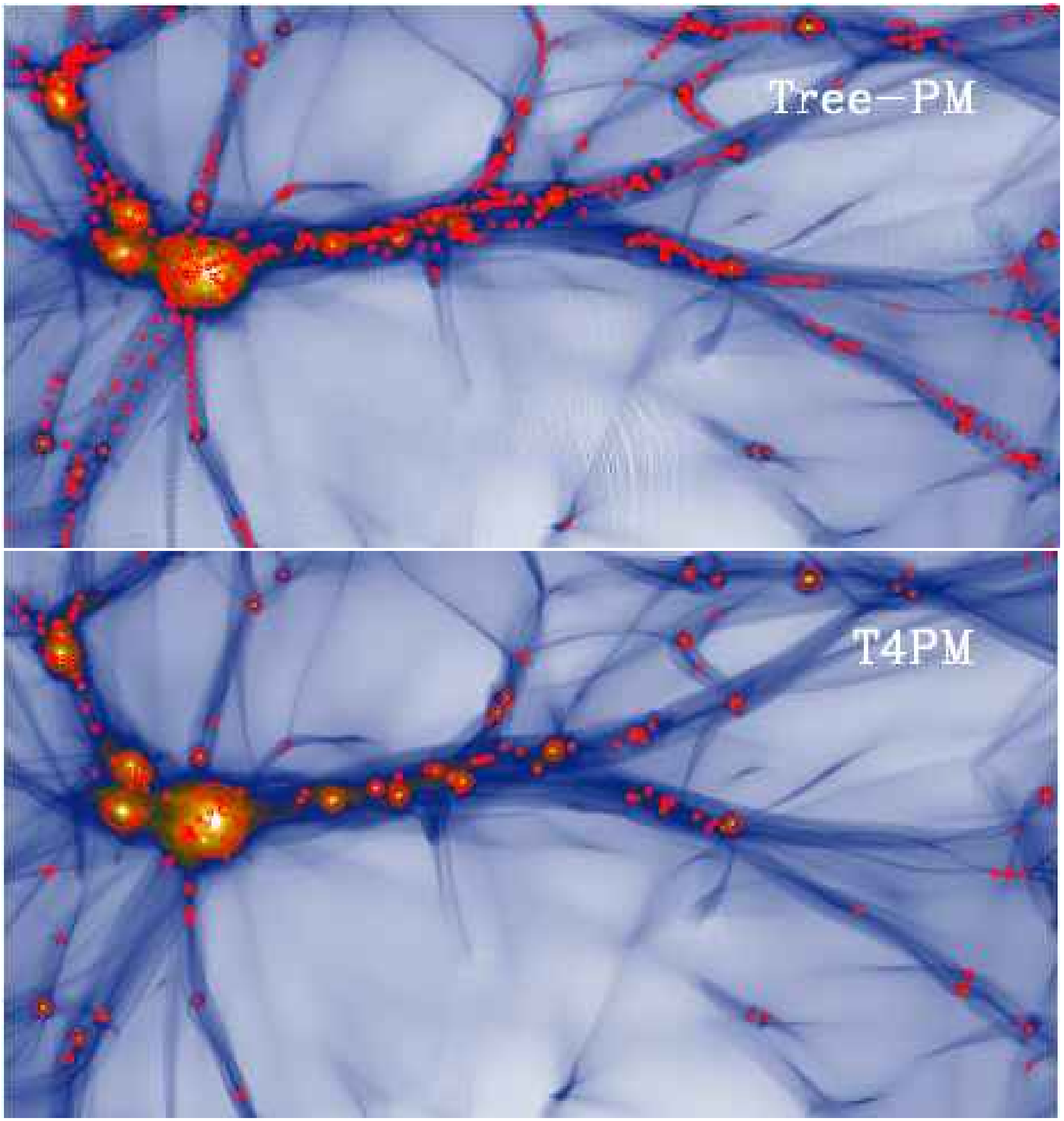
Warm Dark Matter structure formation without noise (Angulo, Hahn, Abel 2013b)

New sheet-based simulation code with reduced collisionality and noise

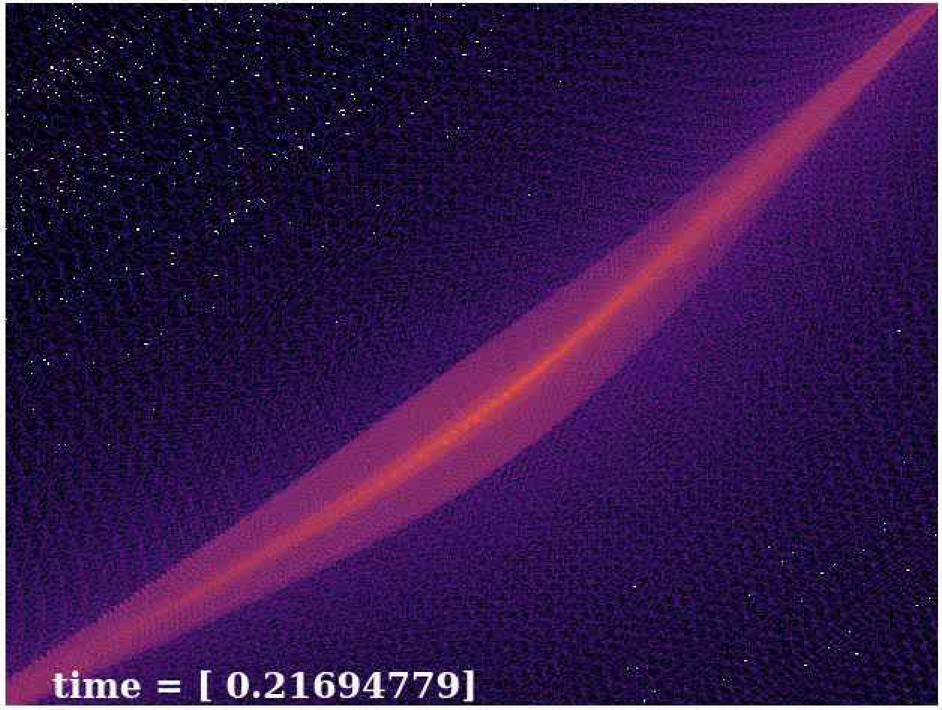


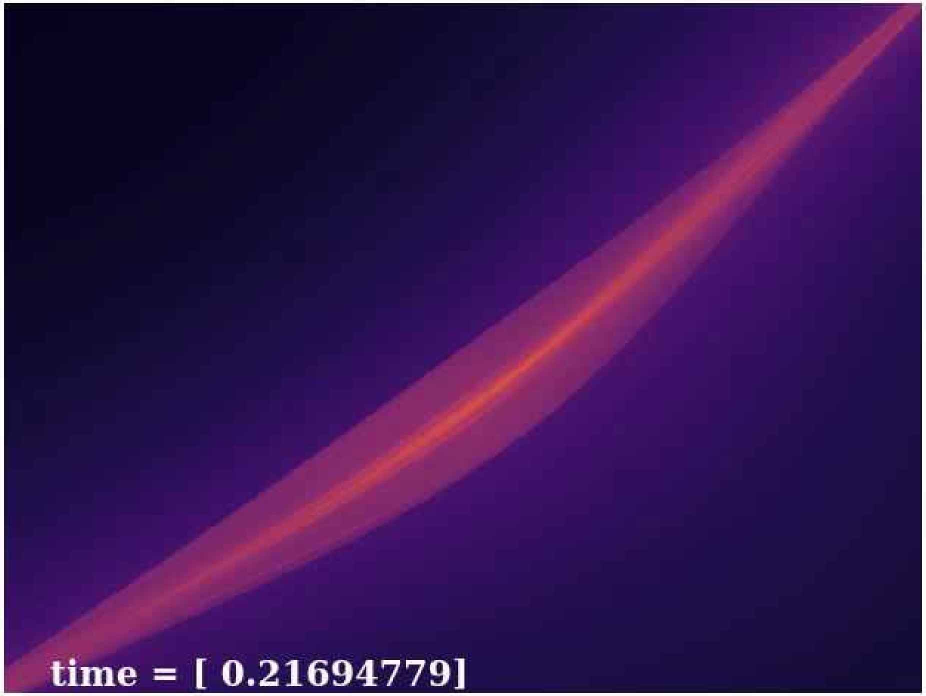
$$\nabla^2 \Phi = \frac{4\pi G}{a} \int f \mathrm{d}^3 v$$

(No need for a “softening length”)



Self-gravitating filament plus spherically-symmetric top-hat perturbation

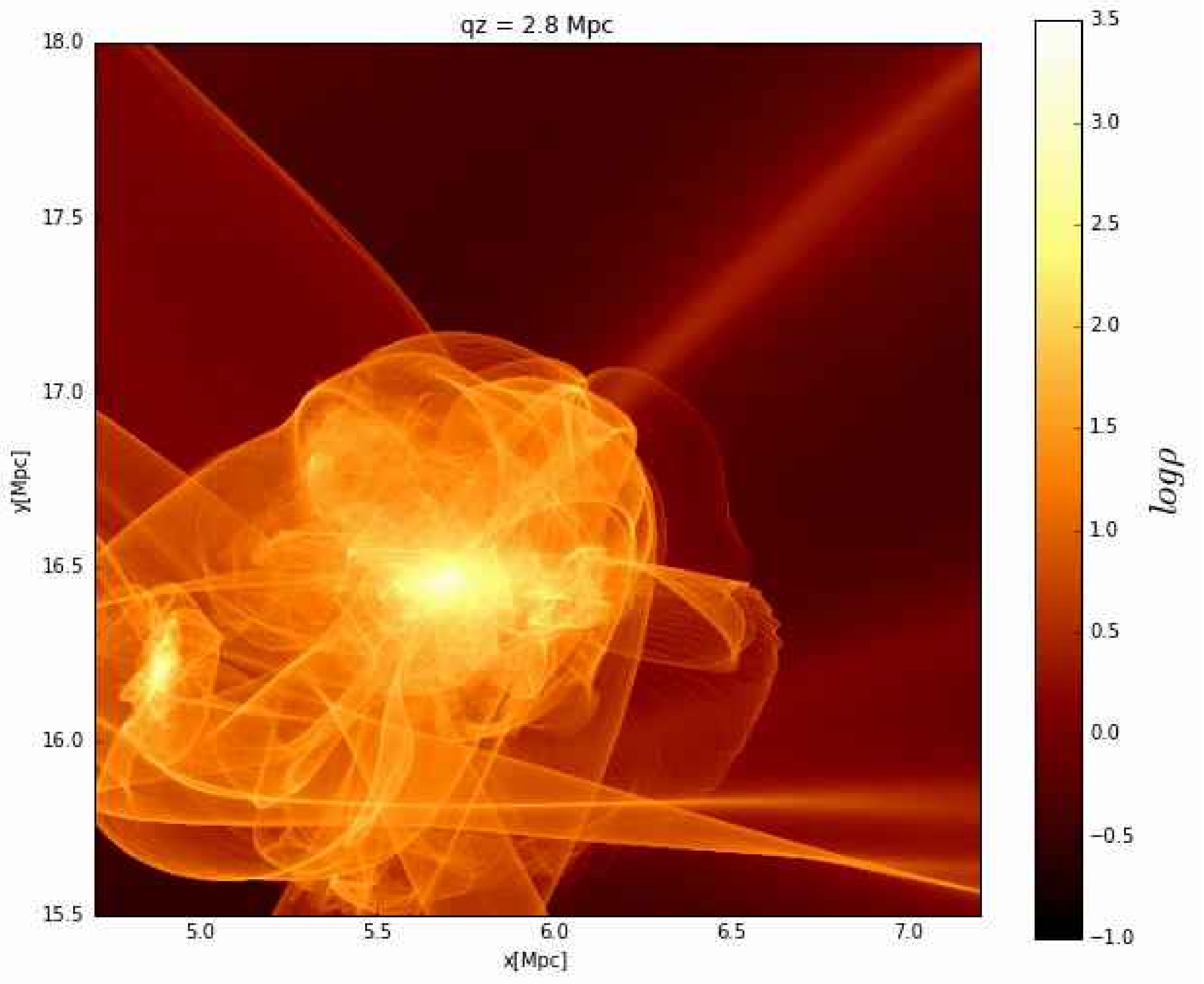




Standard N-body Simulation

Adaptively refined Lagrangian maps

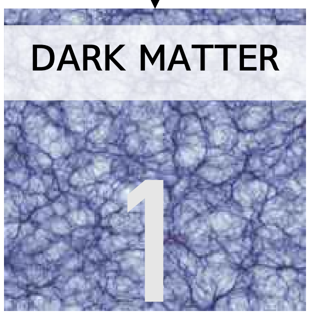
Tesselation of phase-space can recover the full phase-space distribution function



Credit: Jens Stuecker

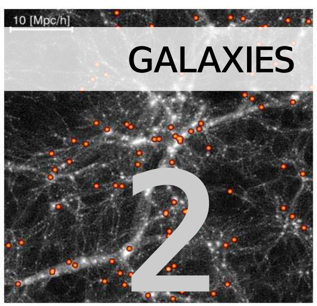
Optimally exploiting future and current surveys is a hard problem

Input Cosmology



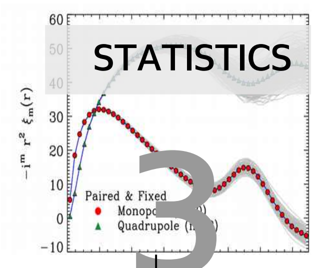
Perturbation theory

- PT breaks quickly
- Higher order expansions loose predictive power



Analytic function

- Galaxy formation physics cannot be fully captured



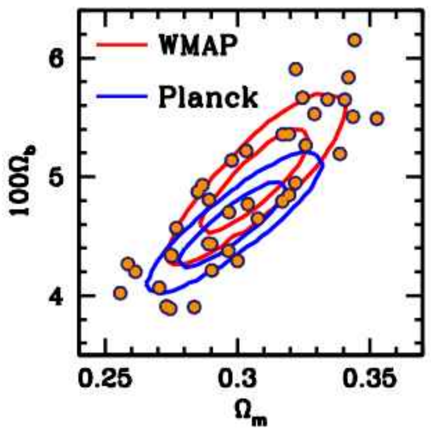
Correlation functions

- Limited set of observables
- Hard to model survey setup
- Unknown likelihoods

Cosmology

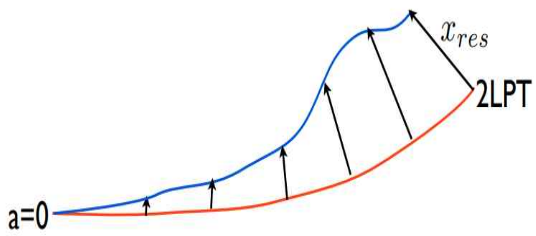
Forward-Modelling LSS observations

Forward: predict observables for a given cosmology.
Backward: infer the cosmology from observables



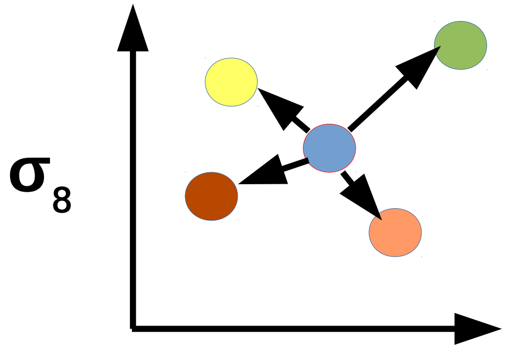
Emulators

Run an ensemble of simulations, compute observables, and interpolate



Approximate N-body

Modify equations of motion for fast solutions



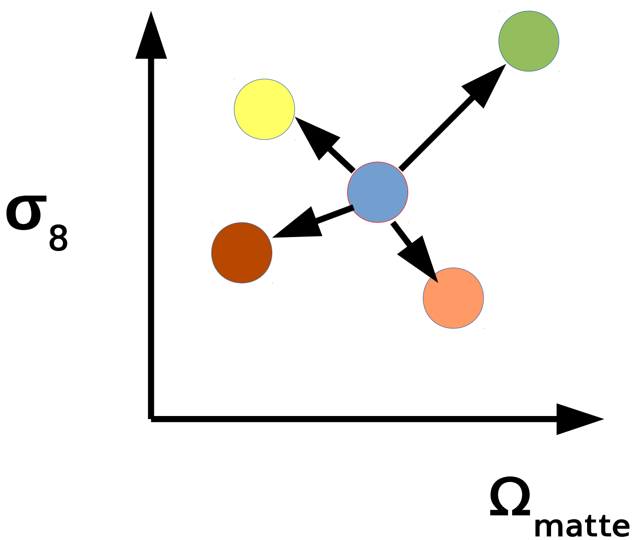
Rescaling Methods

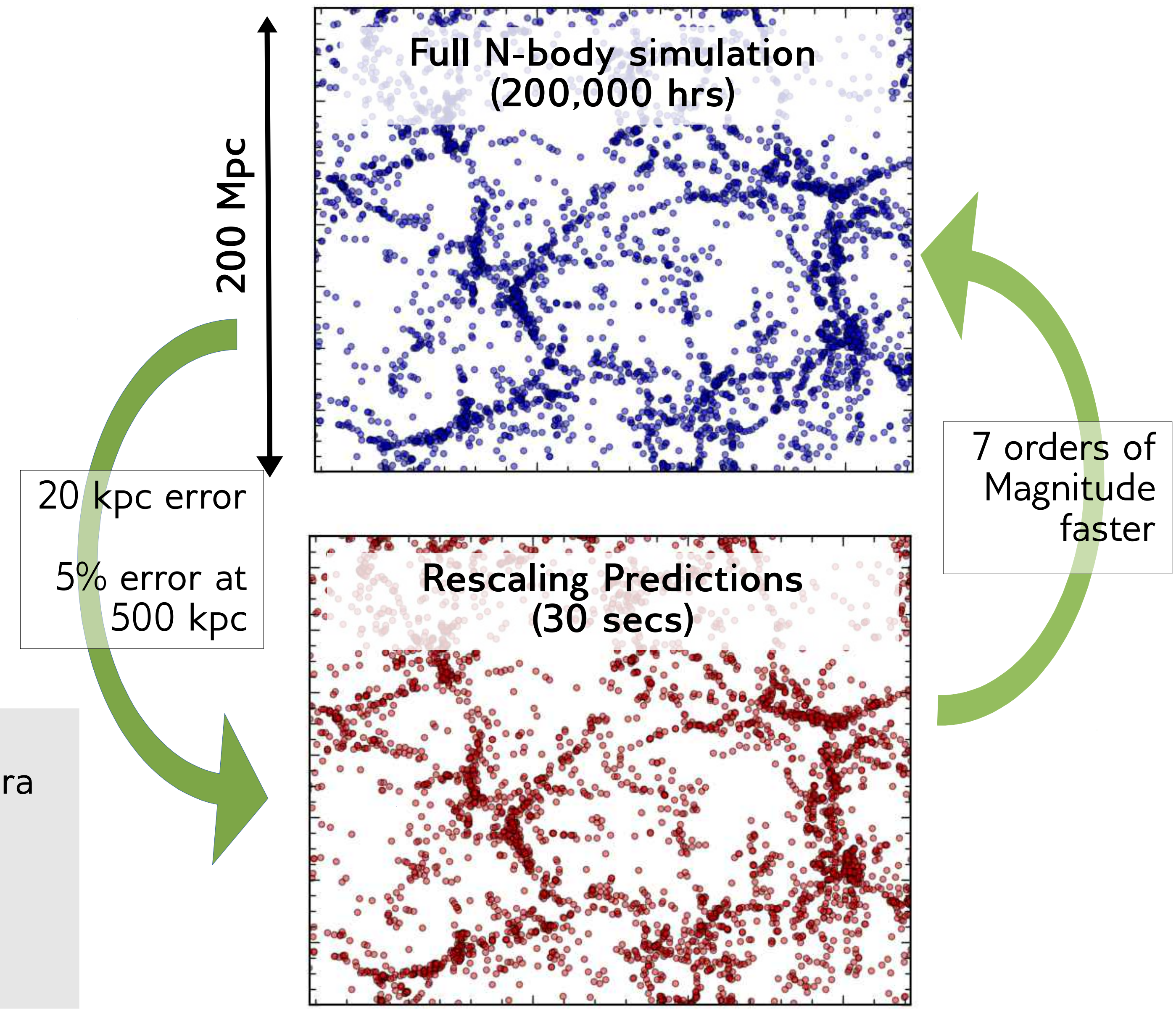
Modify the outputs of a high-res simulation to mimic growth in other cosmologies

Cosmology-rescaling methods

Modify the outputs of simulation to mimic other cosmologies

Angulo & White (2010)
Angulo & Hilbert (2015)



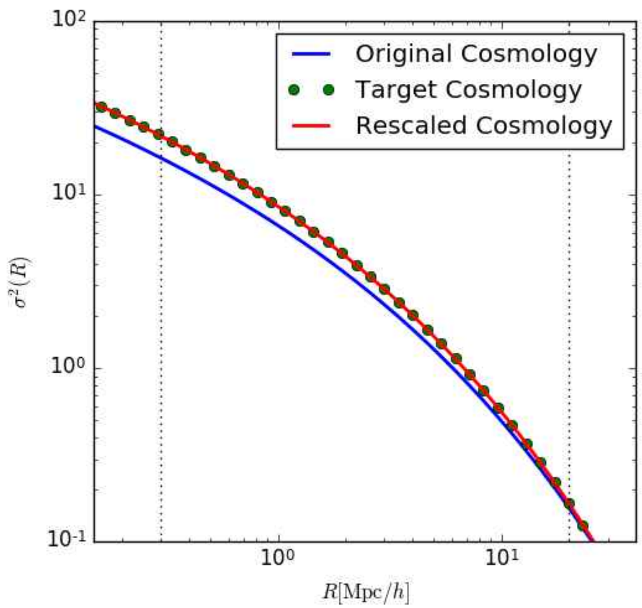


Validated for:

- real/redshift-space
- correlation function/power spectra
- 3-point correlation functions
- (sub)halo mass function
- abundance of voids
- different redshifts/cosmologies

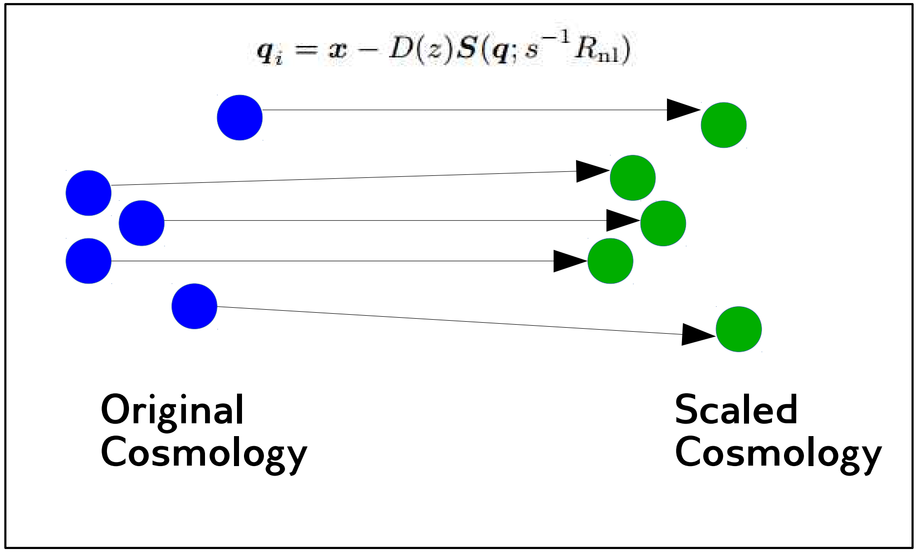
How dœs the cosmology-scaling algorithm works?

Step I:
Length and Time units are changed to match the shape and growth of sigma(R)



Nonlinear Structures

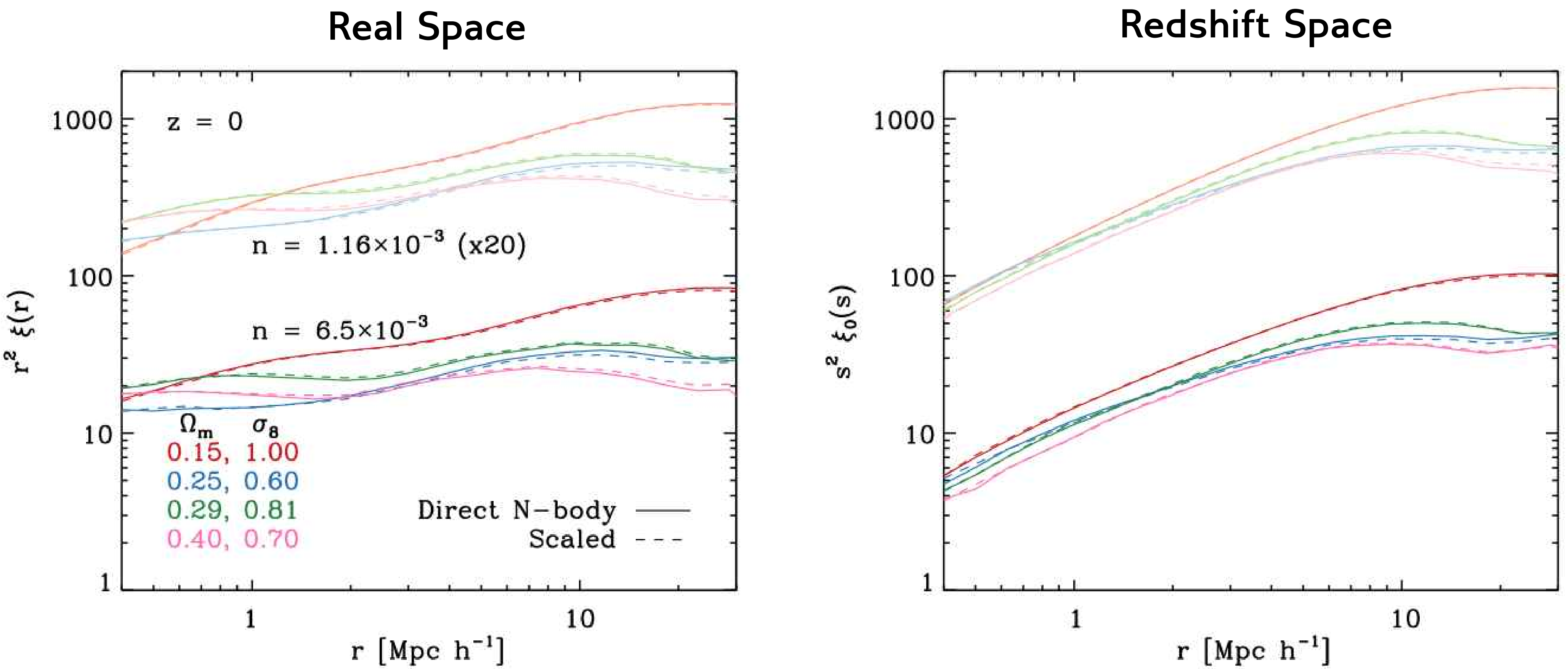
Step II:
Individual large-scale Fourier Modes are modified using the Zel'dovich approximation



Large-scale power spectrum

Error in the scaling algorithm

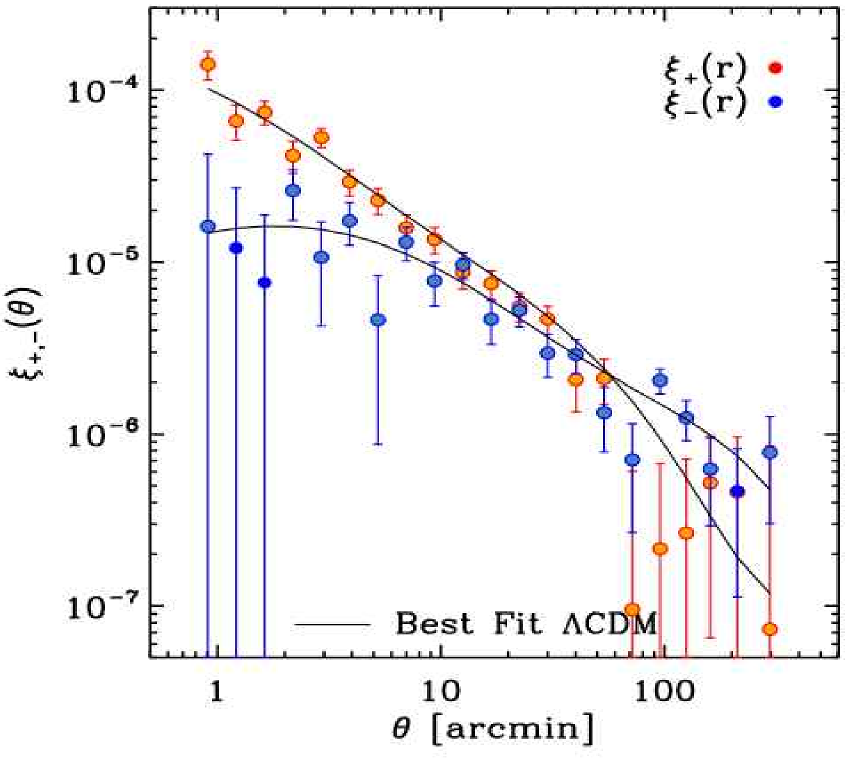
Monopole of the correlation function os DM substrcutures is predicted at 5% level, down to 500 kpc



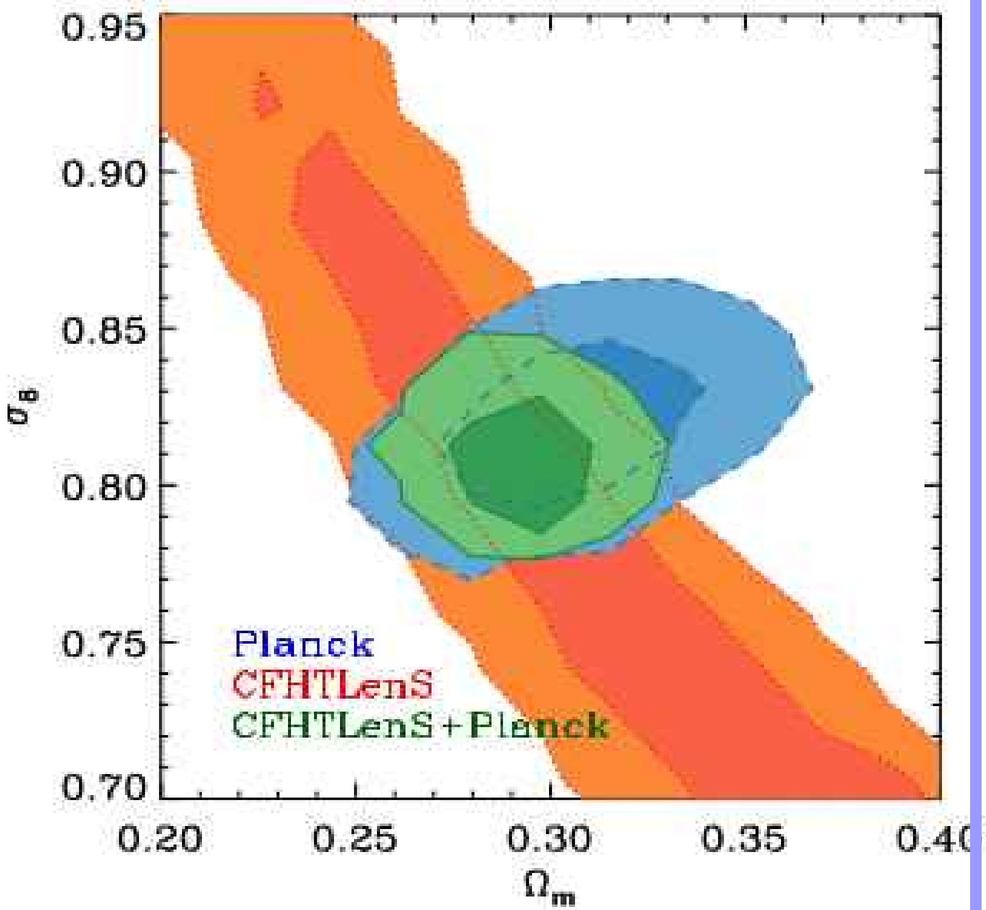
LSS forward modelling applied to lensing

ANALYSIS OF CFHTLenS USING MILLIONS OF SIMULATED UNIVERSES

Shear Correlation measurements



$$\Omega_m = 0.29 \pm 0.01$$
$$\sigma_8 = 0.81 \pm 0.01$$



Angulo & Hilbert 2015

Open Problems & Challenges

Observations are ahead of theory, how can we catch up?

→ Resolve all the host galaxies of future surveys, over volumes larger than those observed

→ How can we increase the accuracy and precision of N-body Simulations?

→ We have a reasonably accurate theory of galaxy formation and nonlinear structures, but it is computationally slow… How do we take advantage of this in cosmological inferences?

Open Problems and Challenges

Can we resolve the full hierarchy of dark matter structures?

→ **Maybe, after 2050...**

→ Resolve the kinematic of stars in the smallest dwarf galaxies

→ What is the origin of nonlinear density profiles?

→ Improved predictions for the phase-space structure

→ Improved modelling of the microphysical properties of DM (and neutrinos).

Open Problems and Challenges

The impact of hydrodynamics/galaxy formation

→ What are the degeneracies between galaxy formation and Cosmology? How can we break those?

→ Under what conditions do baryons affect the central density of galaxies, and the orbits/dynamical friction of galaxies? (i.e. when gravity-only break?)

→ How realistic are current implementations of stellar/AGN feedback (hydrodynamical decoupling, energy injection) of what happens in molecular clouds? (Better treatment of radiation/non-thermal pressure support, non resolved turbulence, etc.)

→ How can we mimic hydrodynamical effects on DM?

Open Problems and Challenges

How to efficiently use the next generation of supercomputer facilities?

Future supercomputers will have ~10,000,000 CPUs, little memory per node, and enhanced by co-processors/GPUs.

→ Future codes will need different parallelisation strategies, have some redundancy, and mixed algorithms.

→ Analysis will be impossible in postprocessing. We need to inline everything in runtime.

→ Data products will be huge… how to best handle and distribute it?