Dark energy models: Theory and observational constraints

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Outline

- Introduction: The standard cosmological model and dark energy models
- Effects of considering dark enery in the geometry and structure growth
- Cosmological probes of dark energy: Supernovae, BAO, CMB, Weak Lensing, Galaxy Clusters
- Constraints on the dark energy equation of states from observational data
- Quintessence models: review and bound from observational data
- K-essence models
- Interacting dark energy models

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The standard cosmological model

The Einstein-Hilbert action can be written as:

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}(R+2\Lambda) + S_M \tag{1}$$

which results in Einstein's equations with cosmological constant

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2}$$

The Friedmann Equations are:

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} \tag{3}$$

and

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{4}$$

For the cosmological constant $p_\Lambda = -\rho_\Lambda = -\frac{\Lambda}{8\pi G}$ and $\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -1$

Dark energy

The effect of dark energy on the expansion rate can be described by:

$$\Omega_{\mathsf{de}} \equiv \frac{\rho_{\mathsf{de},0}}{\rho_{\mathsf{crit},0}} \;; \qquad w \equiv \frac{p_{\mathsf{de}}}{\rho_{\mathsf{de}}} \;. \tag{5}$$

where $\rho_{\mathsf{crit}} \equiv 3H^2/(8\pi G)$ Using the continuity equation for an arbitrary equation of state we can write:

$$\rho_{\mathsf{de}}(z) = \rho_{\mathsf{de},0} \, \exp\left[3\int_0^z \frac{1+w(z')}{1+z'}dz'\right] \tag{6}$$

The expansion rate of the universe $H \equiv \dot{a}/a$ can then be written as

$$\begin{aligned} H^2(z) = H_0^2 \Big[&\Omega_m(1+z)^3 + \Omega_r(1+z)^4 \\ &+ \Omega_{\mathsf{de}}(1+z)^{3(1+w)} + \Omega_k(1+z)^2\Big] \;, \end{aligned} \tag{7}$$

where H_0 is the Hubble constant, Ω_m and Ω_r are the matter and radiation energy densities relative to the critical density.

Energy densities in the Universe

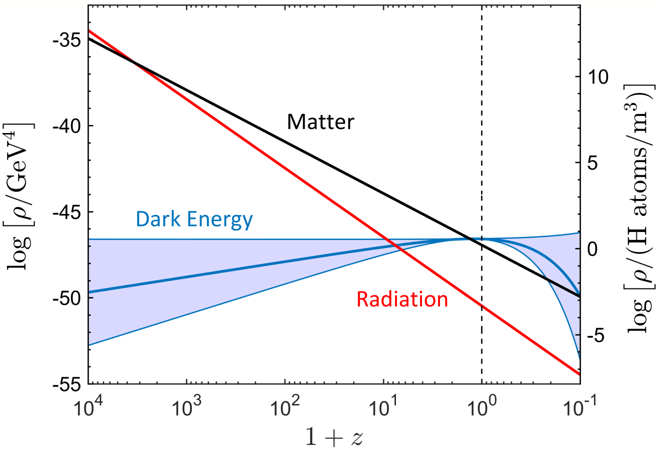


Figure: The shaded region for dark energy indicates the energy densities allowed by combined constraints on current data assuming $\omega(a) = \omega_0 + \omega_a\,(1-a)$. Huterer & Shafer RPP 81, 016901 (2018)

Reconstructed equation of state

$$\omega = \omega_0 + \omega_a(1-a) = \omega_a \frac{z}{1+z}$$

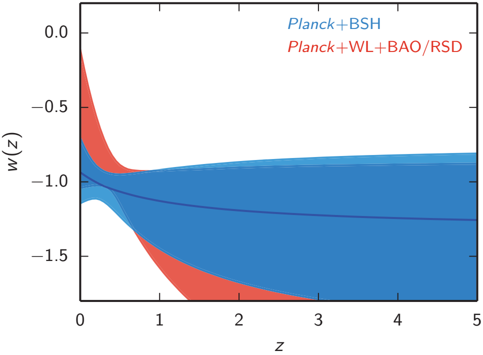


Figure: Planck Collaboration A&A 594, A14 (2016)

Dark energy and geometry

The comoving distance can be written compactly as

$$r(z) = \lim_{\Omega'_k \to \Omega_k} \frac{c}{H_0\sqrt{\Omega'_k}} \sinh\left[\sqrt{\Omega'_k}\int_0^z \frac{H_0}{H(z')}\,dz'\right], \tag{8}$$

The luminosity distance d_L is the distance at which an object with a certain luminosity produces a certain flux $(f = L/4\pi d_L^2)$:

$$d_L(z) = (1+z)\,r(z) \tag{9}$$

The angular diameter distance d_A is the distance at which a certain physical separation $x_{\rm trans}$ produces a certain angle on the sky $(\theta = x_{\rm trans}/d_A)$

$$d_A(z) = \frac{1}{1+z} r(z) \tag{10}$$

Comoving distance

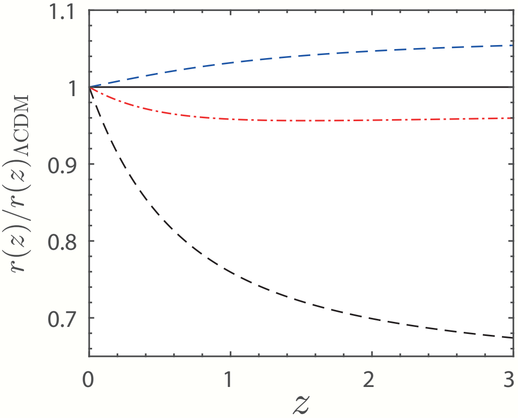


FIGURE: Black Line: Λ_{CDM} model with $\Omega_m = 0.3$, Blue line: Λ_{CDM} model with $\Omega_m = 0.25$, Red line: $\Omega_m = 0.3$ and $\omega = -0.8$, Black dashed line: EdS. Huterer & Shafer RPP 81, 016901 (2018)

Dark energy and growth of matter fluctuations

Asuming $\delta \equiv \delta\rho_m/\rho_m << 1$ on lenght scales much smaller than the Hubble radius: the temporal evolution of the fluctuation can be written as:

$$\ddot{\delta_k} + 2H\dot{\delta_k} - 4\pi G\rho_{\mathsf{m}}\delta_k = 0 \;, \tag{11}$$

where δ_k is the Fourier component corresponding to the mode with wavenumber $k \simeq 2\pi/\lambda$. Let us define de growth suppresion factor $g(a)$ as follows:

$$D(a) \equiv \frac{\delta(a)}{\delta(1)} \equiv \frac{a\,g(a)}{g(1)}. \tag{12}$$

Growth Suppression

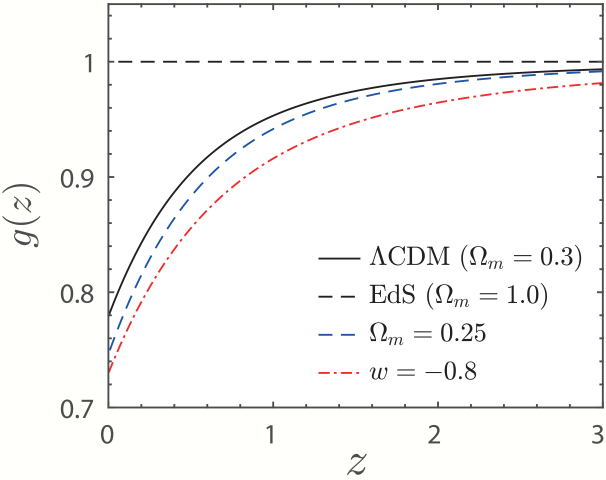


Figure: Huterer & Shafer RPP 81, 016901 (2018)

Type Ia supernovae

We recall the definition of luminosity distance $d_L(z)$:

$$d_L = \sqrt{\frac{L}{4\pi f}} \tag{13}$$

where L is the observed luminosity and f the observed flux. The observational quantity can be written as:

$$m_i - M = 5\log_{10}\left(\frac{d_L}{10\ \mathrm{pc}}\right), \tag{14}$$

$$m_i + \alpha s_i - \beta C_i - \mathcal{M} = 5\log_{10}\left(\frac{d_L}{10\ \mathrm{pc}}\right), \tag{15}$$

where m_i, s_i and C_i are the observed peak magnitude, stretch and color respectively andα, β and $\mathcal{M}$ are the nuissance parameters.

Baryon Acoustic Oscillations

The sound horizon can be computed as the comoving distance that the sound waves could travel from the Big Bang until recombination:

$$r_s = \int_0^{t_*} \frac{c_s}{a(t)}\, dt = \frac{c}{\sqrt{3}} \int_0^{a_*} \frac{da}{a^2 H(a)\sqrt{1 + \frac{3\Omega_b}{4\Omega_\gamma}\, a}} \tag{16}$$

where $a_* \sim 10^{-3}$ is the scale factor at recombination. The galaxy correlation function can be expressed as:

$$\xi(s) = \left\langle \frac{\delta\rho}{\rho}(\mathbf{x_1}) \frac{\delta\rho}{\rho}(\mathbf{x_2}) \right\rangle$$

where ρ and $\delta\rho$ are the mean density and perturbation of matter and $\langle ... \rangle$ is the mean such that que $|\mathbf{x_1} - \mathbf{x_2}| = s$.

Correlation function vs comoving separation

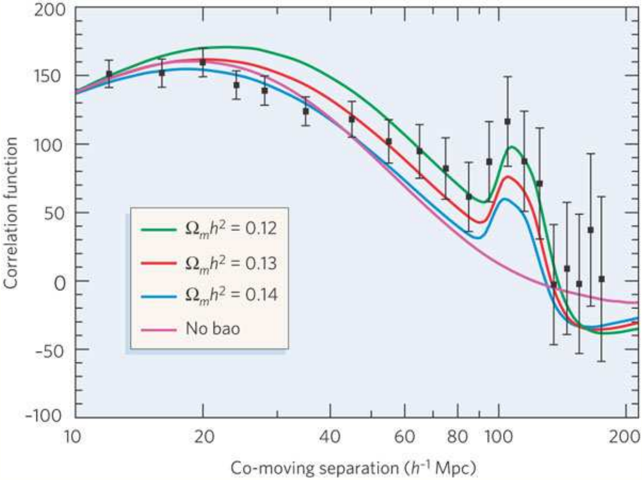


FIGURE: Eisenstein & Bennet Physics Today 61, 44 (2008)

Baryon Acoustic Oscillations

The BAO method measures the cosmic distance scale using the acoustic length scale r_s as a standard ruler.
Separations along the line of sight correspond to differences in redshift

$$\Delta z_s = \frac{H(z)\, r_s}{c}. \tag{17}$$

Separations transverse to the line of sight correspond to differences in angle

$$\Delta\theta_s = \frac{r_s}{d_A(z)}. \tag{18}$$

Up until recently, the BAO measurements had sufficiently large statistical error that it was a good approximation to constrain:

$$D_V(z) \equiv \left[(1+z)^2 d_A(z) \frac{cz}{H(z)}\right]^{1/3}. \tag{19}$$

Dark energy affects the distance to the epoch of recombination, and therefore the angular scale at which the CMB fluctuations are observed. The sound horizon r_s is projected to angle

$$\theta_* = \frac{r_s(z_*)}{r(z_*)}\,, \tag{20}$$

where z_* is the recombination redshift and r is the comoving distance. The CMB essentially constrains the comoving distance to recombination with the physical matter density $\Omega_m H_0^2$ fixed :

$$R \equiv \sqrt{\Omega_m H_0^2}\; r(z_*)\ , \tag{21}$$

The CMB probes a different combination of dark energy parameters than Supernovae or BAO.

Anisotropy of the temperature in the CMB

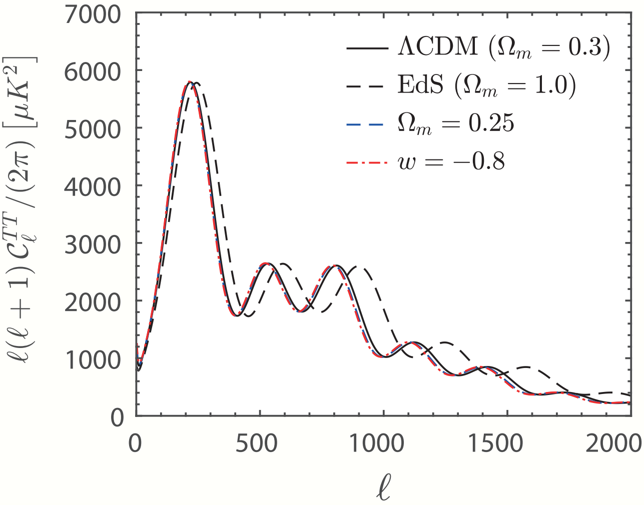


Figure: Huterer & Shafer RPP 81, 016901 (2018)

Galaxy Clusters

The number of halos in redshift interval $[z, z + dz]$ is given by:

$$\frac{dN}{dz}(z) = \int_{4\pi} \frac{dV}{dz d\Omega}(z) \int_0^\infty dM \, \frac{dn(M,z)}{dM} S(M,z) \,, \tag{22}$$

- $\frac{dV}{dz d\Omega}(z) = \frac{r^2(z)}{H(z)}$ is the comoving volume element
- $n(M,z)$ is the galaxy clusters mass function
- $S(M,z)$ is the selection function of the survey

Important concern: how to relate the observable quantity to the mass of the cluster

Predicted cluster counts for a survey

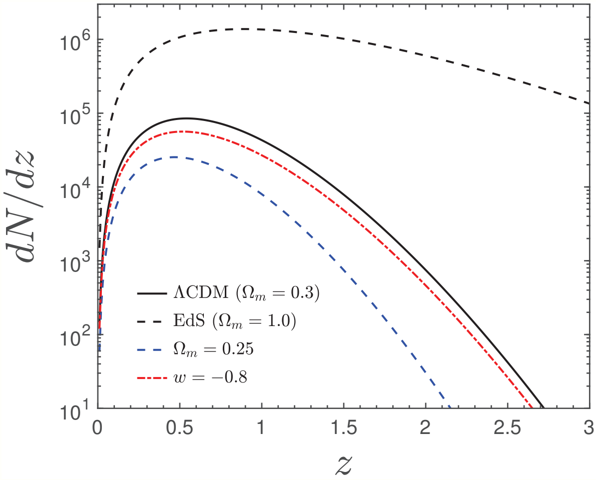


Figure: Huterer & Shafer RPP 81, 016901 (2018)

Weak Lensing

Gravitational lensing produces distortions of images of background galaxies. These distortions can be described as mapping between the source plane (S) and image plane (I),

$$\delta x_i^S = A_{ij} \delta x_j^I \,, \tag{23}$$

where $\delta \mathbf{x}$ are the displacement vectors in the two planes and A is the distortion matrix,

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} . \tag{24}$$

The deformation is described by the convergence κ and complex shear (γ_1, γ_2); the total shear is defined as $|\gamma| = \sqrt{\gamma_1^2 + \gamma_2^2}$.
Given a sample of sources with known redshift distribution and cosmological parameter values, the convergence and shear can be predicted from theory.

Weak Lensing

The convergence can be transformed into multipole space $\kappa_{lm} = \int d\hat{\mathbf{n}}\, \kappa(\hat{\mathbf{n}}, \chi)\, Y^*_{lm}(\hat{\mathbf{n}})$, and the power spectrum is defined as the two-point correlation function $\langle \kappa_{\ell m} \kappa_{\ell' m'} \rangle = \delta_{\ell \ell'}\, \delta_{m m'}\, P^\kappa_\ell$. The convergence angular power spectrum is

$$P^\kappa_\ell(z_s) = \int_0^{z_s} \frac{dz}{H(z) d_A^2(z)} W(z)^2 P\left(k = \frac{\ell}{d_A(z)}; z\right), \qquad (25)$$

where ℓ denotes the angular multipole, $d_A(z)$ is the angular diameter distance, the weight function $W(z)$ is the efficiency for lensing a population of source galaxies and is determined by the distance distributions of the source and lens galaxies, and $P(k, z)$ is the usual matter power spectrum.

Angular power spectrum of cosmic shear

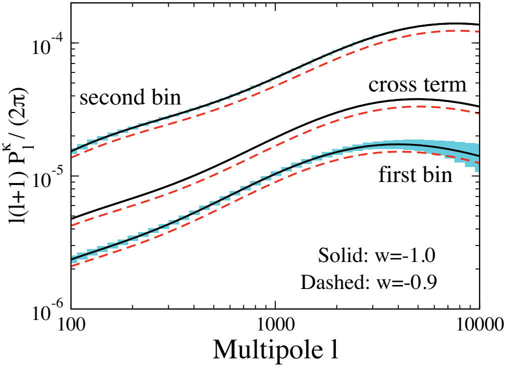


Figure: $0 < z_s < 1$ (first bin) and $1 < z_s < 3$ (second bin) Huterer & Shafer RPP 81, 016901 (2018)

Cosmological tests of dark energy

Probe/Method	Strengths	Weaknesses
SN Ia	pure geometry, model-independent, mature	calibration, evolution, dust extinction
BAO	pure geometry, low systematics	requires millions of spectra
CMB	breaks degeneracy, precise, low systematics	single distance only
Weak lensing	growth & geometry, no bias	measuring shapes, photometric-z biases
Cluster counts	growth & geometry, X-ray, SZ, & optical	mass-observable, selection function

Constraints on ω

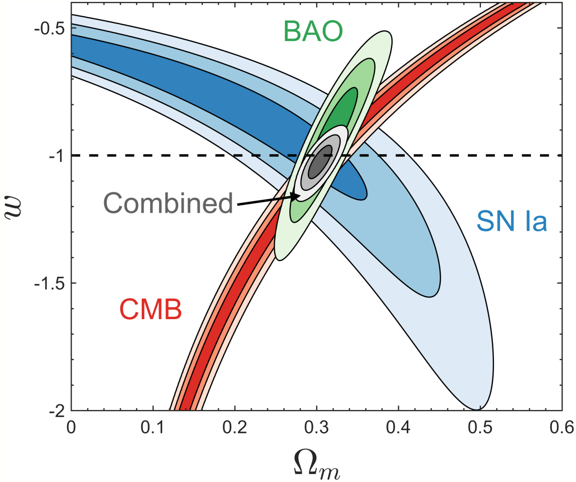


Figure: Huterer & Shafer RPP 81, 016901 (2018)

Constraints on ω

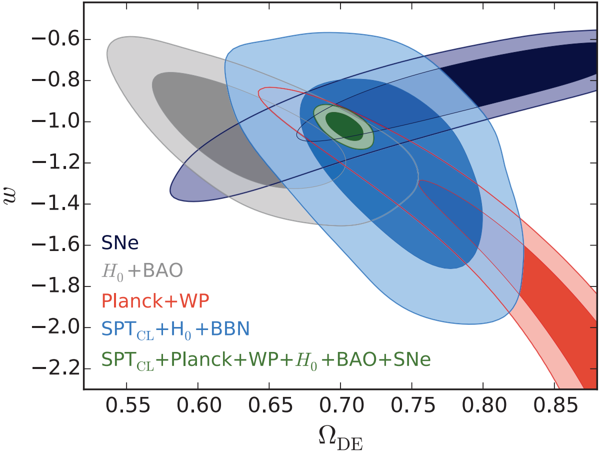


FIGURE: De Haan et al ApJ 832, 95 (2016).

Constraints on ω

$$\omega = \omega_0 + \omega_a(1-a) = \omega_a \frac{z}{1+z}$$

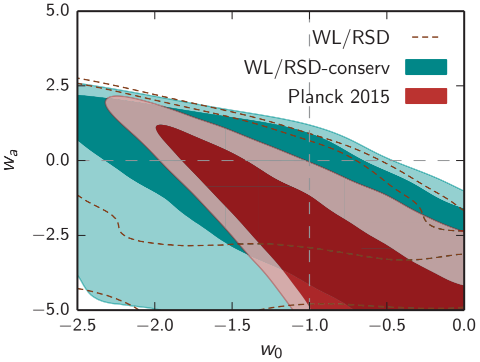


Figure: Joudaki et al 2017

Constraints on ω

$$\omega = \omega_0 + \omega_a(1-a) = \omega_a \frac{z}{1+z}$$

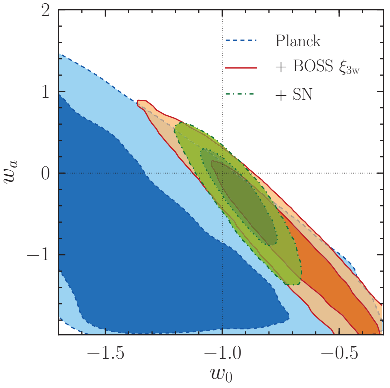


Figure: Sanchez et al 2017

Reconstructed equation of state

$$\omega = \omega_0 + \omega_a(1-a) = \omega_a \frac{z}{1+z}$$

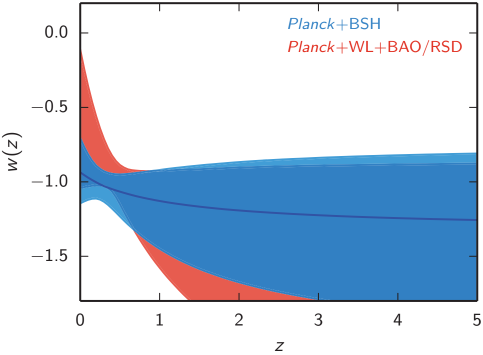


Figure: Planck Collaboration A&A 594, A14 (2016)

Problem of the standard cosmological model

The two problems discussed in the literature can be summarized as follows:

- Fine-Tunning: A big discrepancy between the value of Λ inferred from cosmological observations and the one inferred from thereotical calculations using Quantum Field Theory .
  - The vaccum energy estimated from QFT yieds:

$$\rho_{\rm vac} \simeq 10^{74}\ \mathrm{GeV}^4$$

while cosmological observations give the following value

$$\rho_{\rm DE}^{(0)} \simeq 10^{-47}\,\mathrm{GeV}^4$$

- Cosmic Coincidence: Why is $\rho_m \sim \rho_\Lambda$ today ?

Quintessence models

The action of quintessence is described by

$$S = \int \mathrm{d}^4x\sqrt{-g}\,\left[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] + S_M\,, \qquad (26)$$

where R is a Ricci scalar, and ϕ is a scalar field with a potential $V(\phi)$. The continuity equation, $\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$, translates to the equation of motion of the fiels as follows:

$$\ddot{\phi} + 3H\dot{\phi} + \mathrm{d}V/\mathrm{d}\phi = 0\,, \qquad (27)$$

The field equation of state is given by

$$w_\phi \equiv \frac{P_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}\,. \qquad (28)$$

Quintessence models

From the Einstein equations the following equations can be obtained

$$H^2 = \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_m + \rho_r\right], \tag{29}$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\dot{\phi}^2 + \rho_m + \frac{4}{3}\rho_r\right). \tag{30}$$

Quintessence DE is completely characterized by its equation of state $\omega(t)$. Dark Energy physics beyond a canonical scalar field can be probed by searching for an inconsistency between geometry $H(z)$ and growth $\delta(k;z)$

Classification of quintessence models

The differential equations for w and Ω_ϕ are:

$$\omega_\phi' = (\omega_\phi - 1)[3(1+\omega_\phi) - \lambda\sqrt{3(1+\omega_\phi)\Omega_\phi}]\,, \quad (31)$$

$$\Omega_\phi' = -3(\omega_\phi - \omega_m)\Omega_\phi(1-\Omega_\phi)\,, \quad (32)$$

where a prime represents a derivative with respect to $N = \ln a$.
We will discuss three different cases

- Freezing models: The evolution of the field gradually slows down because the potential tends to be shalow at late times.
    - Tracker models
    - Scaling models
- Thawing models: The field is nearly frozen during the early cosmological epoch and its starts to evolve once $m_\phi < H$.

Tracker models

If

$$\Gamma \equiv V V_{,\phi\phi}/V_{,\phi}^2 > 1$$

and nearly constant the solution is called a **tracker**, along which Ω_ϕ increases and hence $\omega_\phi < \omega_m$ and nearly constant:

$$\omega_\phi = \frac{\omega_m - 2\left(\Gamma - 1\right)}{2\Gamma - 1} \qquad (33)$$

For example, let us consider the inverse power-law potential introduced by Steinhard, Wang and Zlatex PRD **59** 123504 (1999)

$$V(\phi) = M^{4+p}\phi^{-p}\,, \qquad (34)$$

A particular case of the latter is the one studied by Ferreira & Joyce PRL **79** 4740 (1997)

$$V(\phi) = M^5\phi^{-1}$$

.

$\omega(a)$ FOR A TRACKER POTENTIAL

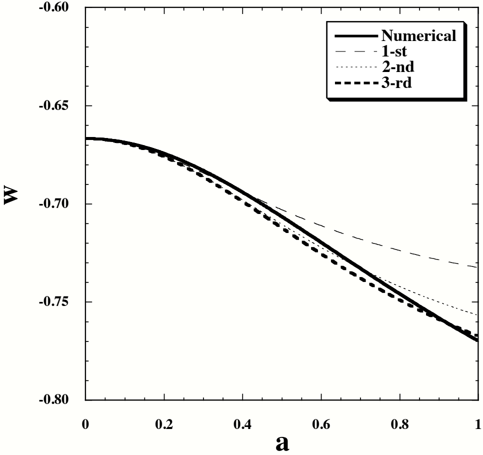


FIGURE: $V(\phi) = M^5\phi^{-1}$ S.Tsujikawa CQG 30 214003 (2013)

Bounds on tracker models from observations

From the joint data analysis of Supernovae (Union2.1), CMB (WMAP7) and BAO (BOSS) data, the tracker equation of state during the matter era is constrained to be with (95% CL)

$$\omega(z = 0) < -0.964$$

under the prior $\omega(z = 0) > -1$ (Chiba et al 2013). For the potential $V(\phi) = M^{4+p}\phi^{-p}$ this bound translates into $p < 0.075$. However, Chiba et al 2013 find that the best-fit corresponds to $\omega = -1$, i.e., the ΛCDM model.

Scaling models

Ferreira & Joyce 1997 proposed

$$V\left(\phi\right) = V_0 e^{\frac{-\lambda\phi}{M_{\rm Pl}}}$$

but this model has some theoretical problems and also can not give a good fit to the observational data. These problems can be alleviated by considering the proposal of Barreiro, Copeland and Nunes PRD **62** 127301 (2000)

$$V(\phi) = V_1 e^{-\lambda_1\phi/M_{\rm pl}} + V_2 e^{-\lambda_2\phi/M_{\rm pl}}\,,$$

The variation of ω_ϕ can be expressed:

$$\omega_\phi(a) = \omega_f + \frac{\omega_p - \omega_f}{1 + (a/a_t)^{1/\tau}}\,,$$

where ω_p and ω_f are asymptotic values of ω_ϕ in the past and future respectively, a_t is the scale factor at the transition between the matter-dominated era and the cosmic acceleration phase, and τ describes the transition width.

$\omega(a)$ FOR A SCALING POTENTIAL

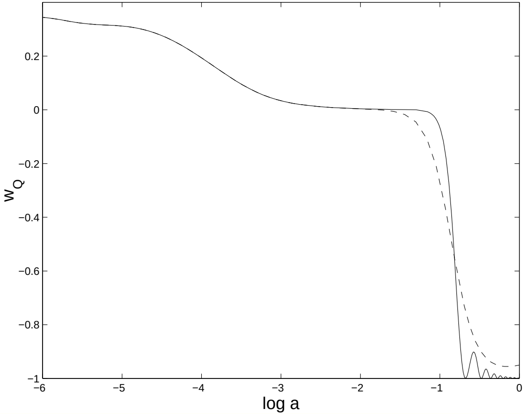


FIGURE: The solid line shows the field equation of state for $\lambda_1 = 20$, $\lambda_2 = 0.5$, while the dashed line shows the case (b) $\lambda_1 = 20$, $\lambda_2 = -20$, $\Omega_\phi = 0.7$.
S.Tsujikawa CQG 30 214003 (2013)

Constraints on scaling models

The joint analysis of Supernovae (Union2.1), CMB (WMAP7) and BAO (SDSS7 and BOSS) data with $\tau = 0.33$ yields the following constraints:

$$a_t < 0.23 \qquad \lambda_1 > 11.7 \qquad \lambda_2 < 0.539$$

An example of these class of models is:

$$V(\phi) = \mu^4 \left[1 + \cos(\phi/f_a)\right] ,$$

where μ and f_a are constants having a dimension of mass. For these kind of models the equation of state is:

$$w(a) = -1 + (1 + w_0) a^{3(K-1)} \mathcal{F}(a) ,$$

where

$$\mathcal{F}(a) = \left[\frac{(K - F(a))(F(a) + 1)^K + (K + F(a))(F(a) - 1)^K}{(K - \Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2} + 1)^K + (K + \Omega_{\phi 0}^{-1/2})(\Omega_{\phi 0}^{-1/2} - 1)^K} \right]^2 .$$

and

$$K \equiv \sqrt{1 - \frac{4}{3} \frac{M_{\rm pl}^2 V_{,\phi\phi}(\phi_i)}{V(\phi_i)}} , \qquad F(a) \equiv \sqrt{1 + [(\Omega_{\phi 0})^{-1} - 1] a^{-3}} ,$$

$\omega(a)$ FOR A THAWING POTENTIAL

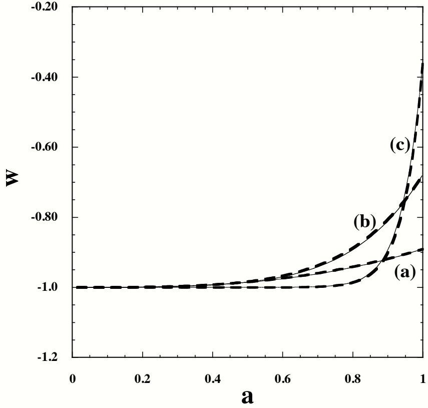


FIGURE: The field equation of state w with (a) $f_a/M_{\rm pl} = 0.5$, $\phi_i/f_a = 0.5$ ($K = 1.9$), (b) $f_a/M_{\rm pl} = 0.3$, $\phi_i/f_a = 0.25$ ($K = 2.9$), and (c) $f_a/M_{\rm pl} = 0.1$, $\phi_i/f_a = 7.6 \times 10^{-4}$ ($K = 8.2$). $\Omega_{\phi 0} = 0.73$. S.Tsujikawa CQG 30 214003 (2013)

Allowed region for Freezing and Scaling models

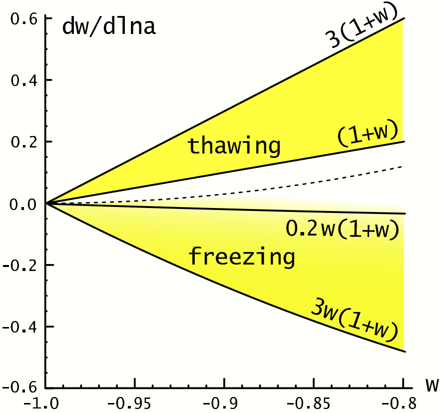


FIGURE: S.Tsujikawa Astrophysics and Space Science Library 370 p. 331(2011) arXiv:1004.1493

Quintessence parametrized models

Acoording to Huang et al 2011 we can parametrize ω at late times:

$$w = -1 + \frac{2}{3}\epsilon_S F^2\left(\frac{a}{a_{\rm de}}\right) , \quad (35)$$

where ϵ_S is defined as:

$$\epsilon_S \equiv \epsilon_V\big|_{a=a_{\rm de}} , \quad (36)$$

with $\epsilon_V \equiv (\frac{d\ln V}{d\phi})^2 M_{\rm pl}^2/2$ being a function of the slope of the potential, $a_{\rm de}$ is the scale factor where the total matter and DE densities are equal. The function $F(x)$ is defined as:

$$F(x) \equiv \frac{\sqrt{1+x^3}}{x^{3/2}} - \frac{\ln\left(x^{3/2}+\sqrt{1+x^3}\right)}{x^3} . \quad (37)$$

Eq.35 is only valid for late-Universe slow-roll ($\epsilon_V \lesssim 1$ and $\eta_V \equiv M_{\rm pl}^2 V''/V << 1$) or the moderate-roll ($\epsilon_V \lesssim 1$ and $\eta_V \lesssim 1$) regime.

CONSTRAINTS ON PARAMETRIZED QUINTESSENCE MODELS

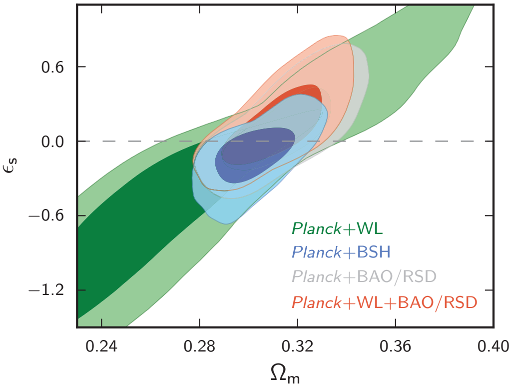


FIGURE: Planck Collaboration A&A 594, A14 (2016)

For quintessence models, where the scalar field rolls down from a very steep potential, at early times $\epsilon_V(a) \gg 1$, however the fractional density $\Omega_\phi(a) \to 0$ and the combination $\epsilon_V(a)\Omega_\phi(a)$ aprroaches a constant, defined to be a second parameter:

$$\epsilon_\infty \equiv \lim_{a \to 0} \epsilon_V(a)\Omega_\phi(a) \tag{38}$$

While ϵ_S is sensitive to the late time evolution of $1 + \omega(a)$, ϵ_∞ captures its early time behaviour.

Constraints on parametrized quintessence models

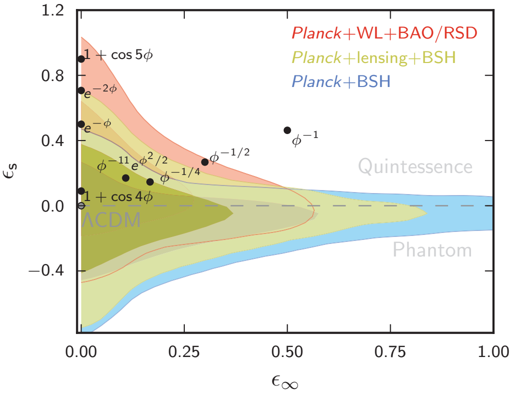


Figure: Planck Collaboration A&A 594, A14 (2016)

K-essence models

The full action including a k-essence term is given by (Armendariz-Picon 2000)

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi G}R + p(\phi, X)\right] + S_M, \tag{39}$$

where $X \equiv \frac{1}{2}(\nabla\phi)^2$ is the canonical kinetic energy of the field, and $p(\phi, X)$ plays a role of the pressure p_K. Here we consider a model with

$$p_K = p(\phi, X) = \tilde{p}(X)/\phi^2, \tag{40}$$

which has the desired property for dark energy. For small X, $\tilde{p}(X)$ could be expanded as $\tilde{p}(X) = const. + X + \mathcal{O}(X^2)$. If we ignore the non-linear term $\mathcal{O}(X^2)$ and take an additional potential, then we come back to the quintessence model. **The scalar field for which these higher order kinetic energy terms play an essential role is k-essence**.

K-essence models

The energy density of the k-field is

$$\rho_K = (2X\tilde{p}_{,X} - \tilde{p})/\phi^2 \equiv \tilde{\rho}/\phi^2 \quad (41)$$

so that the equation of state parameter for the k-field is:

$$\omega_K \equiv \frac{p_K}{\rho_K} = \frac{\tilde{p}}{\tilde{\rho}} = \frac{\tilde{p}}{2X\tilde{p}_{,X} - \tilde{p}} \quad (42)$$

If p satisfies the condition $Xp_{,X} \ll p$ for some range of X and ϕ, then the equation of state is $p \approx -\rho$. The effective speed of sound c_s of k-essence is defined by:

$$c_s^2 = \frac{p_{,X}}{\rho_{,X}} = \frac{\tilde{p}_{,X}}{\tilde{\rho}_{,X}}. \quad (43)$$

The Friedmann equations for a flat FRW space-time :

$$H^2 \equiv \dot{N}^2 = \frac{8\pi G}{3}(\rho_M + \rho_K), \qquad N \equiv \ln a. \tag{44}$$

Using the energy conservation equation and considering a homogeneous field ϕ, we get

$$\frac{dX}{dN} = -\frac{\tilde{\rho}}{\tilde{\rho}_{,X}}\left[3(1+\omega_K) - 2\phi^{-1}\frac{\sqrt{2X}}{H}\right]. \tag{45}$$

Remember that $X \equiv \frac{1}{2}(\nabla\phi)^2$.

It is usual to use a new variable $y \equiv 1/\sqrt{X}$. The equation of state and speed of sound in terms of the new variable:

$$\omega_K = -g/(yg') \qquad c_s^2 = \frac{p'_K}{\rho'_K} = \frac{g - g'y}{g''y^2}, \tag{46}$$

Using this new variable, the stability conditions can be expressed as $g' < 0$ and $g'' > 0$ so that g is a decreasing convex function of y. The equation of motion for the k-field,

$$\frac{dy}{dN} = \frac{3}{2}\frac{(\omega_K(y) - 1)}{r'(y)}\left[r(y) - \sqrt{\frac{\rho_K}{\rho_{tot}}}\right], \tag{47}$$

where

$$r(y) \equiv \left(-\frac{9}{8}g'\right)^{1/2} y(1 + \omega_K) = \frac{3}{2\sqrt{2}}\frac{(g - g'y)}{\sqrt{-g'}}. \tag{48}$$

The attractor solutions for k-essence are classified into two types:

- a tracker solution in which k-essence mimics the equation of state of the background component in the Universe
- k-essence is attracted to an equation of state which is different from matter or radiation.

For all types of attractors there is a set of initial conditions which evolve towards the attractor.

We want to have a tracker solution $y(N)$ which satisfies:

$$\omega_K(y_{tr}) = -\frac{g}{yg'}|_{y=y_{tr}} = \omega_M \tag{49}$$

The point y_{tr} that satisfies those conditions is the so-called attractor solution. In order to get an atractor solution we need that:

$$r^2(y_{tr}) = \frac{\rho_K}{\rho_{tot}} < 1, \tag{50}$$

in the range of $r(y) > 1$ there is no attractor solution.

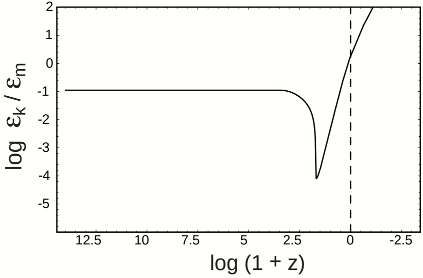
K-ESSENCE ATRACTOR SOLUTIONS

- Trackers
  - Radiation Trackers : $\omega_K(y_{tr}) = \omega_r = \frac{1}{3}$
  - Dust Trackers : $\omega_K(y_{tr}) = \omega_m = 0$
- Atractors
  - De-Sitter atractors: $\omega_K(y_{tr}) = -1$
  - K-atractors: $\omega_K(y_{tr}) < 0$

There are two possible scenarios: In both of them, first the k-essence field is attracted to $y = y_R$ in the radiation dominated epoch. The, at matter dominated epoch, ρ_K drops sharply by several orders of magnitude

- The k-essence field is not atracted to the dust atractor during the matter dominated era. In the following the field is atracted to $y \approx y_S$, ρ_K freezes and overtakes ρ_m. And then y relaxes towards y_K. In this scenario, our current Universe lies on the transition from y_S to y_K.
- After matter domination, the k-essence approaches first the S-attractor, freezes for a finite time, is attracted towards the dust attractor, and the Universe decelerates its expansion. This scenario is called *late dust tracker* because the dust attractor is reached long after the matter domination has begun.

Ratio of k-essence to matter energy density



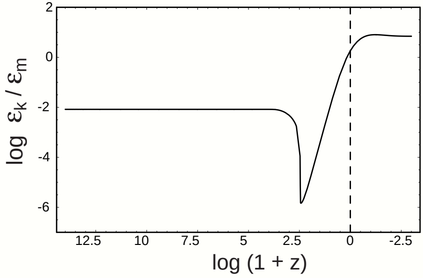
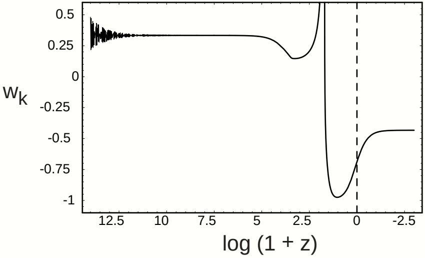


Figure: Left: Model with a k-atractor; Right: Model with a late dust tracker solution; Armendariz-Picon, Mukhanov & Steinhardt PRD 63, 103510 (2001)

Equation of state vs redshift



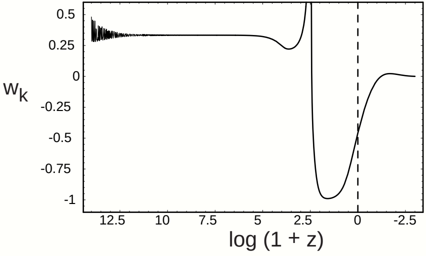


Figure: Left: Model with a k-atractor; Right: Model with a late dust tracker solution; Armendariz-Picon, Mukhanov & Steinhardt PRD 63, 103510 (2001)

- Low energy efective string theory

The action of low energy effective string theory in the presence of a higher-order derivative term $(\tilde{\nabla}\phi)^4$ is given by

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^4\tilde{x}\sqrt{-\tilde{g}} \left[F(\phi)\tilde{R} + \omega(\phi)(\tilde{\nabla}\phi)^2 + \alpha' B(\phi)(\tilde{\nabla}\phi)^4 + \mathcal{O}(\alpha'^2) \right] , \tag{51}$$

Under a conformal transformation, $g_{\mu\nu} = F(\phi)\tilde{g}_{\mu\nu}$, we obtain the action in the Einstein frame:

$$S_E = \int \mathrm{d}^4x\sqrt{-g} \left[\frac{1}{2\kappa^2} R + K(\phi)X + L(\phi)X^2 + \cdots \right] , \tag{52}$$

- Ghost condensate

Let us consider the ghost condensate model characterized by the Lagrangian:

$$P = -X + X^2/M^4\,, \tag{53}$$

where M is a constant. A more general version of this model, called the dilatonic ghost condensate is:

$$P = -X + e^{\kappa\lambda\phi} X^2/M^4\,, \tag{54}$$

which is motivated by a dilatonic higher-order correction to the tree-level action.

- Tachyon

A tachyon field appears as an unstable mode of D-branes. The effective 4-dimensional Lagrangian is given by :

$$P = -V(\phi)\sqrt{1-2X}\,, \tag{55}$$

where $V(\phi)$ is a potential of the tachyon field ϕ. It can be used for dark energy provided that the potential is shallower than $V(\phi) = V_0\phi^{-2}$.

- Dirac-Born-Infeld (DBI) theory

The field dynamics can be described by the DBI action for a probe D3-brane moving in a radial direction of the Anti de Sitter (AdS) space-time. The Lagrangian density with the field potential $V(\phi)$ is given by

$$P = -f(\phi)^{-1}\sqrt{1-2f(\phi)X} + f(\phi)^{-1} - V(\phi)\,, \tag{56}$$

where $f(\phi)$ is a warped factor of the AdS throat.

Quintessence vs k-essence

- In both models the cosmic evolution is insensitive to initial conditions because the field is attracted to the attractor solution wherever it started.
- In both models the the coincidence problem is solved by explaining why the cosmic acceleration is started at such a late stage shortly after the onset of the matter dominated phase.
- Neither quintessence nor k-essence solve the vacuum energy problem.
- For the tracker solution, the quintessence field tracks the radiation and matter background, and needs a potential energy fine-tuning at the quintessence-matter crossover stage.
- The k-essence field tracks only the radiation background (for no D-attractor scenario), and does not need a potential energy term thus it is free from fine-tuning that arose in quintessence.

Coupled dark energy and dark matter

The interaction between dark matter and dark energy is described by following modified energy conservation equations

$$\dot{\rho}_m + 3H(\rho_m) = \delta, \quad (57)$$
$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\delta, \quad (58)$$

where δ is an energy exchange term in the dark sector. There are two major examples:

$$\delta = \kappa Q \rho_m \dot{\phi}, \quad (59)$$
$$\delta = \alpha H(\rho_m + \rho_\phi), \quad (60)$$

where Q and α are dimensionless constants.

Coupled dark energy and dark matter

The first case corresponds to a potential:

$$V(\phi) = V_0 e^{-\kappa\lambda\phi}. \tag{61}$$

The coupled quintessence scalar field equation is

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\kappa Q \rho_m, \tag{62}$$

In the second case the interaction potential and coupling structure are determined from the requirement $\frac{\rho_m}{\rho_\phi} = const.$.
The coupling equation is equivalent to:

$$\dot{\phi}\left[\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}\right] = -\delta, \tag{63}$$

Coupled dark energy and dark matter

For example the model developed by L.Chimento and collaborators:

$$\rho'_{\mathrm{m}} + \gamma_{\mathrm{m}}\rho_{\mathrm{m}} = -Q, \qquad \rho'_{\mathrm{x}} + \gamma_{\mathrm{x}}\rho_{\mathrm{x}} = Q. \tag{64}$$

The equation for the energy density ρ of the dark sector:

$$\rho'' + (\gamma_{\mathrm{m}} + \gamma_{\mathrm{x}})\rho' + \gamma_{\mathrm{m}}\gamma_{\mathrm{x}}\rho = Q(\gamma_{\mathrm{m}} - \gamma_{\mathrm{x}}). \tag{65}$$

Here, the nonlinear interaction Q between both dark components is $Q = \alpha\rho'\rho$, with α being the coupling constant.

Results for coupled models

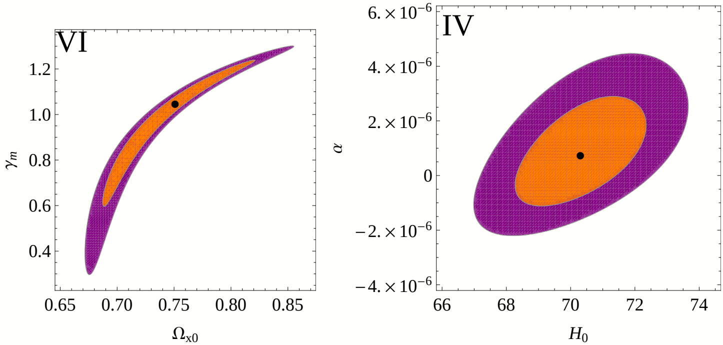


Figure: L.Chimento et al. PRD 88 087301 (2013)

Summary

- Cosmological probes for Dark Energy
  - Supernovae Type Ia
  - Baryon Acoustic Oscillations
  - Cosmic Microwave Background
  - Galaxy Clusters
  - Weak Lensing
- Theoretical Models
  - Quintessence Models
  - k-essence models
  - Coupled dark energy and dark matter

Thank you !!! Obrigado !!! Gracias!!!