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Cosmological Phase Transitions and Gravitational Waves

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Abstract

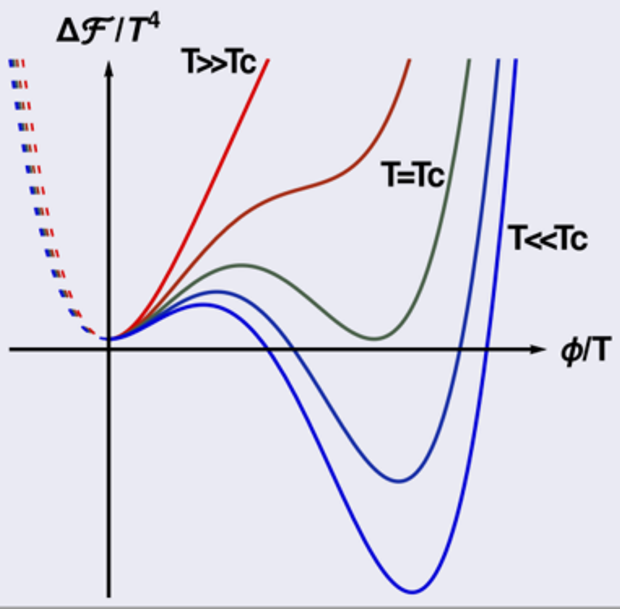
Extensions of the particle physics Standard Model with boson sector strongly coupled to Higgs field predicts Gravitational Waves (GW) signal, generated during the electroweak phase transition, in the LISA detection range. This conclusion results from studying hydrodynamics of the phase transition to calculate fluid profiles, released energy distribution and characteristic sizes involved.

What are Phase Transitions & Gravitation Waves (GW)?

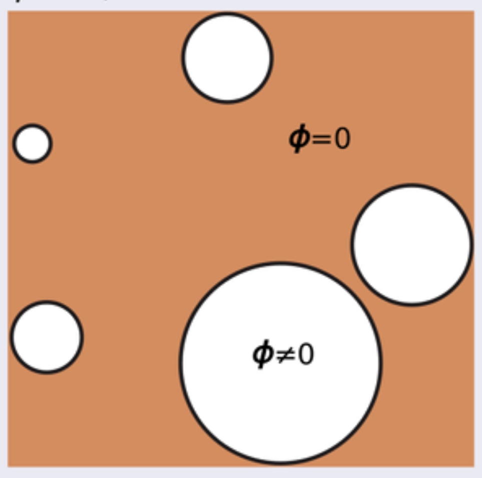
Using *Quantum Field Theory at Finite Temperature* formalism it is possible to calculate Landau free energy $\mathcal{F}$ (thermal effective potential). At high temperature T, it has the form

$$\mathcal{F} = \rho_{fv} - \frac{\pi^2}{90} g_* T^4 + D\left(T^2 - T_0^2\right)\phi^2 - ET\phi^3 + \frac{\lambda(T)}{4}\phi^4$$

Where ϕ is the thermodynamic expectation value of a scalar field (e.g. Higgs field) and g_*, D, $E\ldots$ depend on the number of degrees of freedom, masses and coupling constants of the particle physics model.



At very high temperatures there is only a minimum $\phi = 0$; at critical temperature T_c there are two minima with equal value of $\mathcal{F}$; for $T \ll T_c$ a phase transition occurs and bubbles begins to nucleate. Outside the bubbles, the universe is in the phase with $\phi = \phi_+ = 0$ (*the "+" phase*, which dominates at high temperature). Inside the bubble, the universe is in the phase with $\phi = \phi_- \neq 0$ (*the "−" phase*, which dominates at low temperature).



For the electroweak phase transition $\phi \neq 0$ represents the broken symmetry phase (inside the bubbles, the particles gain mass thanks to the Higgs mechanism). Due to the jump of free energy $\mathcal{F}(\phi_-) < \mathcal{F}(\phi_+)$ there is a difference of pressure ΔP between phases ($P = -\mathcal{F}$), then the bubbles grow and primordial plasma is perturbed by the bubble wall advance.

The GW are ripples in spacetime, more precisely

- The spacetime metric is discomposed, $g_{\mu\nu} = g_{\mu\nu}^{(B)} + h_{\mu\nu}$
  - $g_{\mu\nu}^{(B)}$ is low frequency background metric (*Friedmann*).
  - $h_{\mu\nu} = \Delta g_{\mu\nu}$ is high frequency and small amplitude perturbation.
- *Einstein eq*, $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, linearization leads to a wave equation:

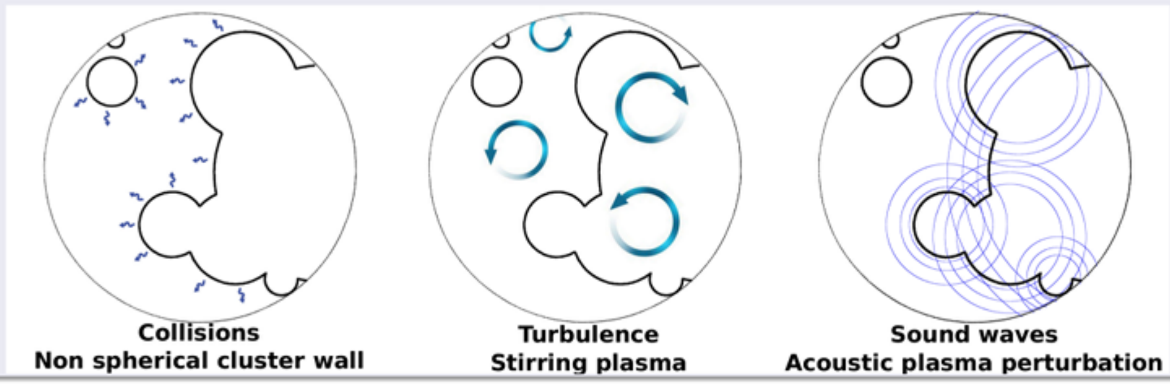
$$\nabla^2 \tilde{h}_{ij} - a^2 \partial_t^2 \tilde{h}_{ij} - 3a\dot{a}\partial_t \tilde{h}_{ij} = -16\pi G a^2 \Delta \tilde{T}_{ij}$$

$\tilde{h}_{ij}$ is projection of the spatial part of $h_{\mu\nu}$, such that $\partial_i \tilde{h}_{ij} = 0$ and $\tilde{h}_{ii} = 0$

- $g_{\mu\nu}^{(B)}$ and $h_{\mu\nu}$ are coupled, $h_{\mu\nu}$ is "source" of $g_{\mu\nu}^{(B)}$, with energy density

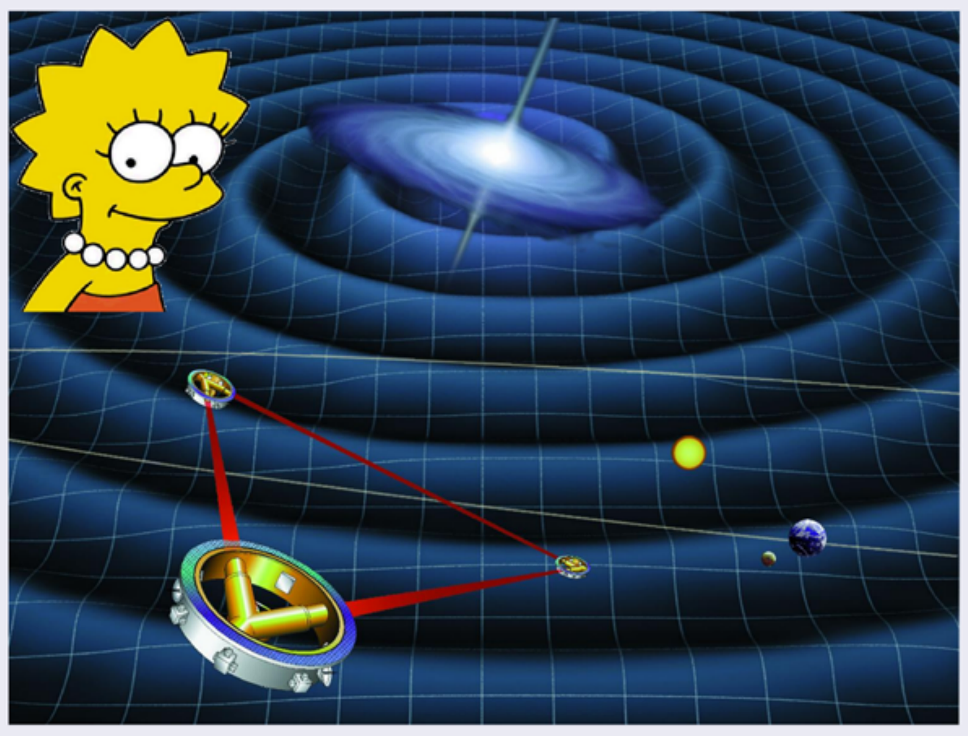
$$\rho_{\rm GW} = \langle \partial_t \tilde{h}_{ij} \partial_t \tilde{h}_{ij} \rangle / (16\pi G) \quad \text{(This is the "gravitational wave energy")}$$

When different bubble walls meet, spherical symmetry disappears. Several GW sources have been proposed:



What is LISA? Why LISA?

It is not Lisa Simpson. It is Laser Interferometer Space Antenna (joint effort between ESA/NASA). LISA will be the first observatory in space to explore the Gravitational Universe. It is designed to detect and accurately measure GW from astronomical sources. But,...is it possible detect cosmological signals (like GW from electroweak phase transtion)?



Radiation era: $e \approx \rho_R \propto gT^4$ y $P \approx e/3 \;\Rightarrow s = \partial P/\partial T \propto gT^3$
Adiabatic expansion: $S \propto sa^3 = cte \Rightarrow a_*/a_0 = (g_0/g_*)^{1/3} T_0/T_*$
"0" for today and "$*$" for then (phase transition time), then

$$a_*/a_0 \approx 8\times10^{-16}\,(100/g_*)^{1/3}\,100\text{GeV}/T_*$$

If the scale of the source is L_S (with $L_S/H^{-1} \sim 10^{-2}$) and $f_* \sim 1/L_S$, for electroweak phase transition ($g_* \sim 100$ y $T_* \sim 100\,$GeV):

$$f_0 \sim 1{,}6\times10^{-5}\,\text{Hz}\left(\frac{g_*}{100}\right)^{1/6}\left(\frac{T_*}{100\,\text{GeV}}\right)\frac{f_*}{H_*} \sim f_*\,a_*/a_0 \sim 10^{-3}\,\text{Hz}$$

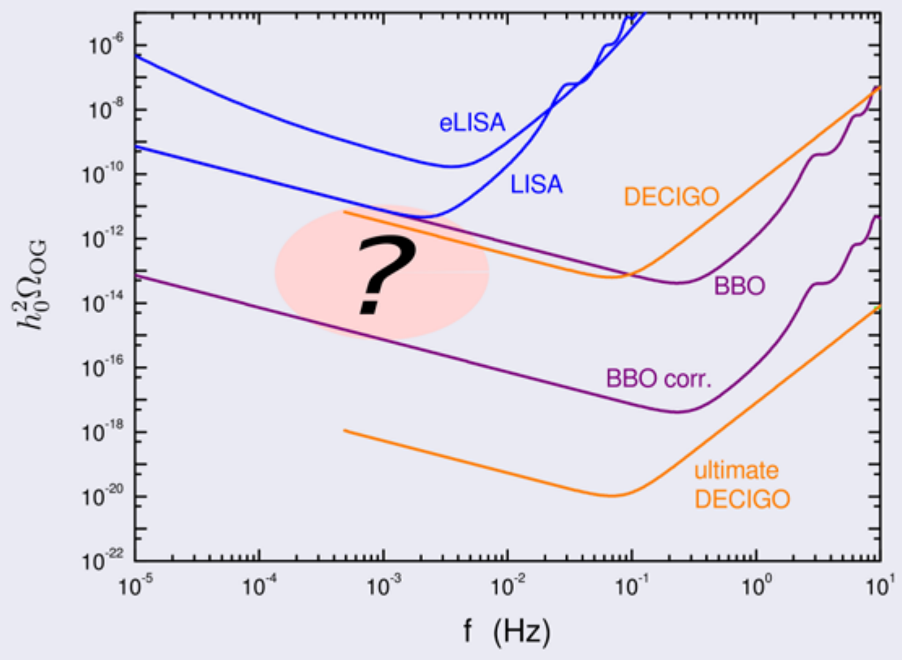
Roughly, the derivative $\partial h \sim h/L_S$
GW source $\Delta T \sim e_{\rm kin}$ (kinetic energy density of fluid)
Wave equation: $\partial^2 h \sim G\,\Delta T \;\Rightarrow\; L_S{}^{-2} h \sim G\,e_{\rm kin} \Rightarrow h \sim L_S{}^2 G\,e_{\rm kin}$
Wave energy density: $\rho_{\rm OG} \sim \partial h \partial h/G \;\Rightarrow\; \rho_{\rm OG} \sim G\,e_{\rm kin}{}^2 L_S{}^2$
Friedmann equation $H^2 \propto G\,e$ implies:

$$\rho_{\rm OG*} \sim (e_{\rm kin}/e_*)^2 (L_S H_*)^2 \rho_{R*}$$

Also, it can be defined $\Omega_{\rm OG} \equiv \rho_{\rm OG0}/\rho_c$ with $\rho_{\rm OG0} = \rho_{\rm OG*}(a_*/a_0)^4$ and $\Omega_R \equiv \rho_{R0}/\rho_c \approx 5\times10^{-5}$, where $\rho_{R0} \propto g_0 T_0^4$ and $\rho_{R*} \propto g_* T_*^4$.
$\Rightarrow\; \rho_{R*}(a_*/a_0)^4 = (g_0/g_*)^{1/3}\rho_{R0}$

Taking $g_0 \sim 3$, $e_{\rm kin} \sim (m_H\,v)^2$ [vacuum energy released, where m_H is the Higgs's mass and $v \equiv \phi_-$ (at $T=0$), the vacuum expectation value], *almost* all energy is $e_* \sim g_* T^4 \sim g_* v^4$ ($\frac{H_*}{T_*} \sim \frac{v}{M_P} \sim 10^{-17}$):

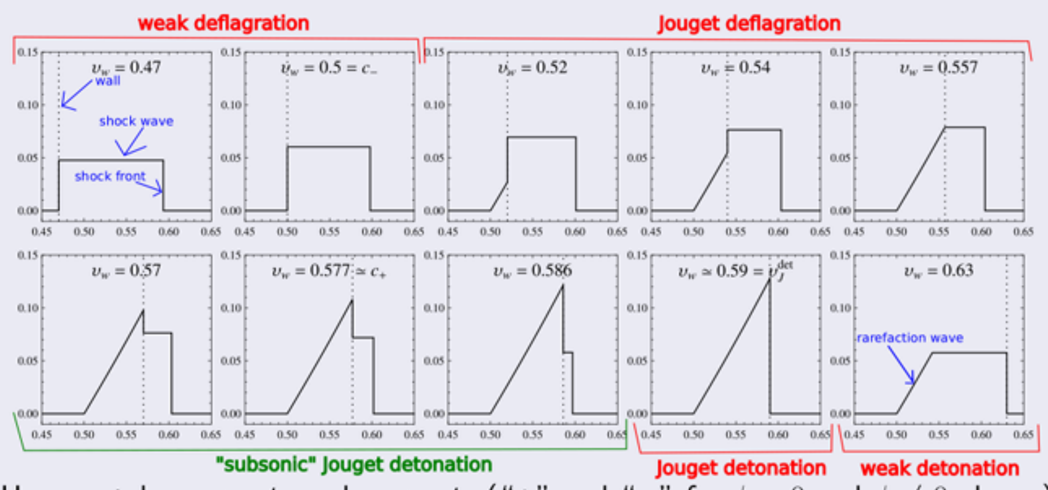
$$\Omega_{\rm OG} \sim 5\times10^{-5}\,(g_0/g_*)^{1/3}\,(e_{\rm kin}/e_*)^2\,(L_S H_*)^2 \sim 10^{-13}$$



The quick estimation suggests detection chances, and even some day other detectors, like DECIGO or BBO, will be much more sensibles.

Hydrodynamics

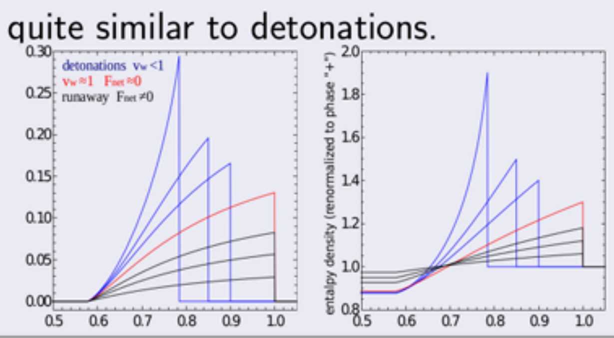
For different wall velocity v_w there is different stationary profiles v vs. r/t



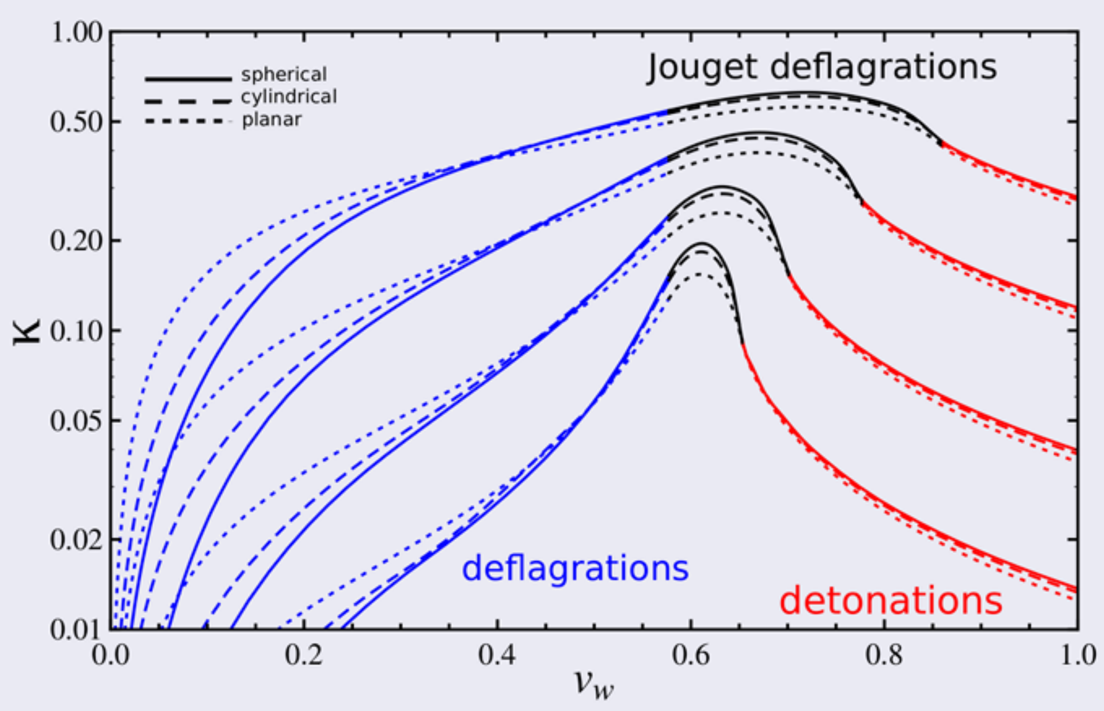
Here we take $c_+ \approx cte$ and $c_- \approx cte$ ("+" and "−" for $\phi = 0$ and $\phi \neq 0$ phases)
Also we use a bubble wall with planar symmetry, but the spherical case is pretty similar

We found the new **"subsonic" Jouget detonations** [the fluid just in front of the wall has a subsonic speed ($< c_+$) relative to the wall]. Specialized bibliography usually uses an approximation to free energy $\mathcal{F}$, the *Bag model* (where $c_+ = c_- = 1/\sqrt{3}$), which hides this solution. We improved the approximation with a model that takes two different speed of sound.

Stationary regime occurs when a *friction force* compensates a *driving force* (like a pressure difference between phases) and the net force is $F_{\rm net} = 0$. But, if it does not happen, the wall accelerates and $v_w \approx 1$. Such *runaway* regime is quite similar to detonations.



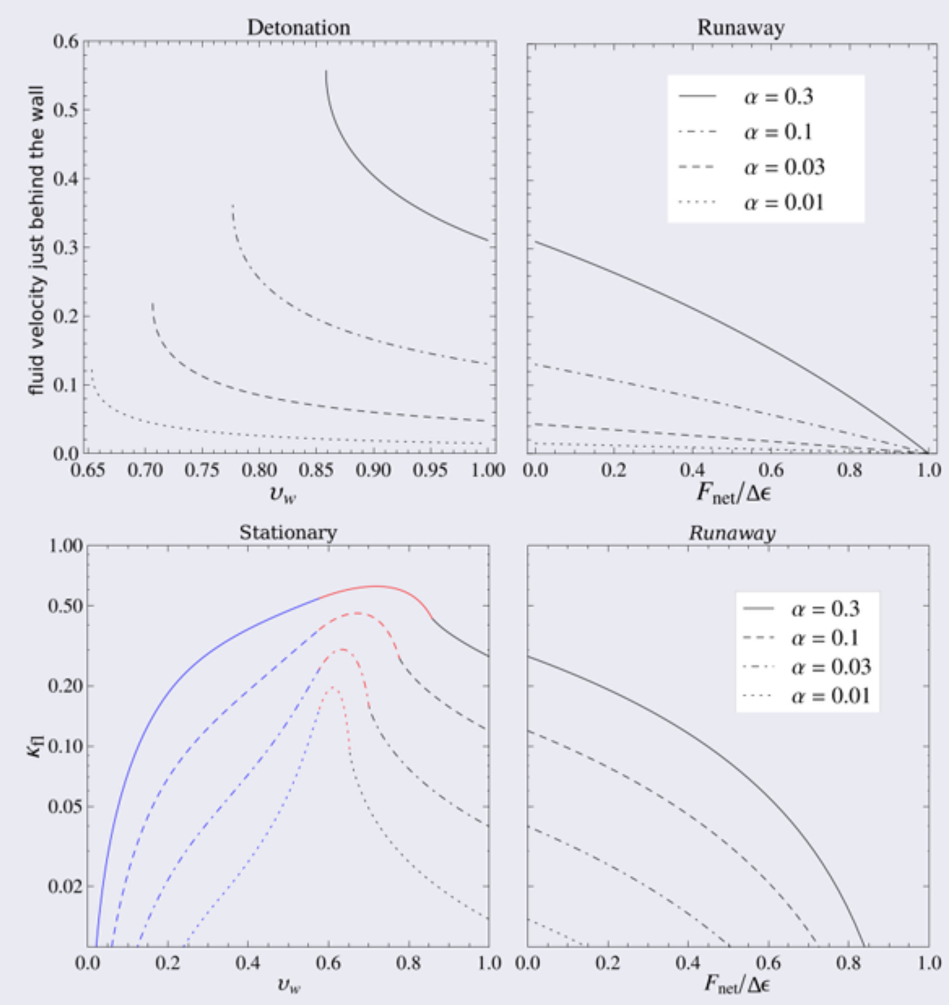
For GW calculation is relevant the *efficiency factor* κ_{fl} and κ_w, the fraction of released energy that goes to fluids motions and bubble wall. We have tested that κ is quite similar for different bubble geometries



In this parametric analysis we use a parameter $\alpha \sim$"realeased false vacuum energy"/"radiation contribution to free energy ($\propto T_+^4$)". From bottom to top $\alpha = 0{,}01$, $0{,}03$, $0{,}1$, $0{,}3$.

We found "semi-analytical" expressions for planar case, which could be use instead of a harder calculation for spherical case, and also we found that *supersonic Jouget deflagrations* have the biggest κ, which implies higher GW amplitude from turbulence and sound waves mechanisms.

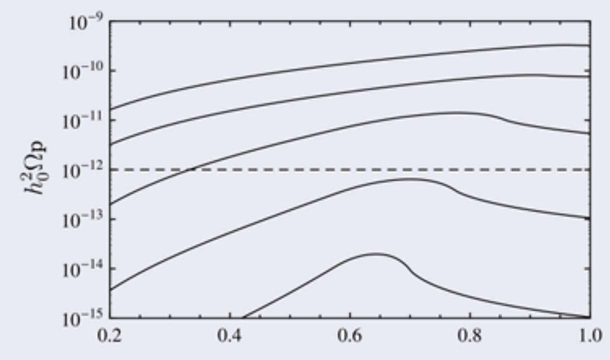
Stationary regime has a negligible κ_w. When the wall is *runaway* it accumulates energy, then the fluid has less perturbance



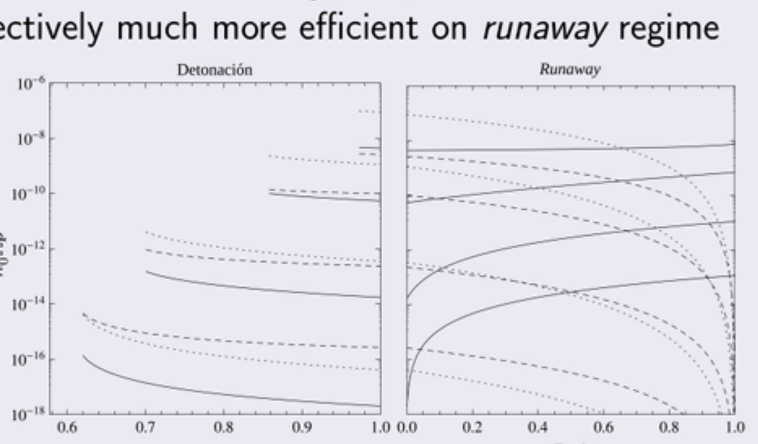
Lower κ_{fl} and higher κ_w implies higher energy concentration in bubble wall and higher GW amplitude from collisions mechanism.

GW spectrum

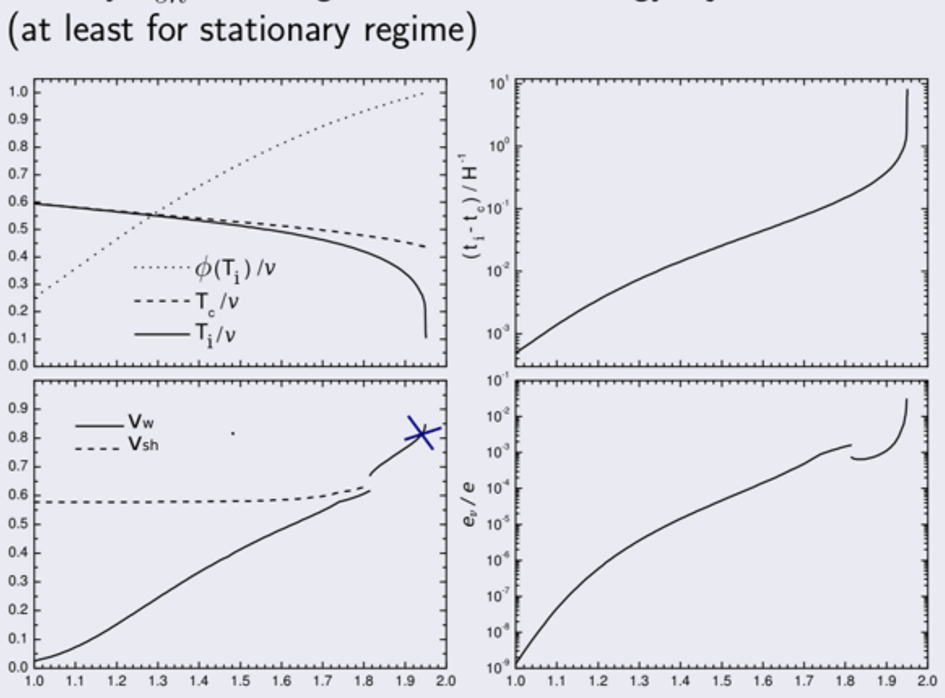
A parametric study for peak amplitude Ω_p by turbulence mechanism. Here deflagrations, Jouget deflagrations, and detonations are covered varying all the entire range of v_w

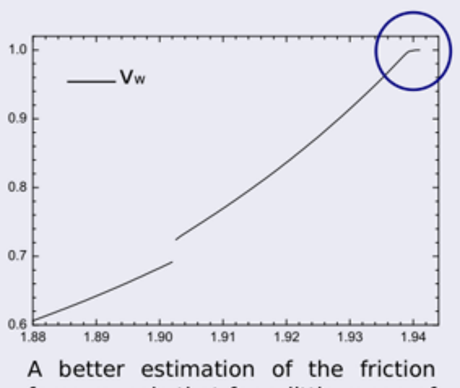


Another parametric study for GW generation by collisions (dashed), turbulence (line) and sound wave (doted) mechanisms. It is noteworthy that, the collisions mechanism is effectively much more efficient on *runaway* regime



We applied the hydrodynamics study to an extension of the Standard Model of particle physics with a sector of bosons coupled to the Higgs field. Higher value of coupling constant h (horizontal axis in figures) implies higher value of the minimum position ϕ_- (and so, by dimensionality, higher free energy barrier), lower critical temperature $T_c = T(t_c)$ and phase transition "initiation" temperature $T_i = T(t_i)$, higher values of wall velocity v_w and shock velocity v_{sh}, and higher fraction of energy injected into bulk motions of the plasma e_v/e (at least for stationary regime)

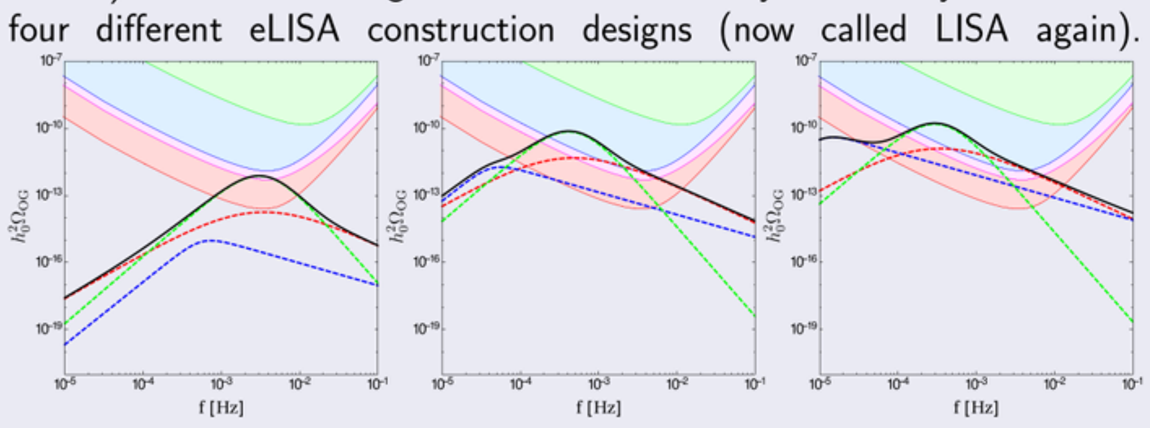




A better estimation of the friction force reveals that for a little range of high values of h there is *runaway*

Here we have considered a number of degrees of freedom $g = 2$ for the scalar sector. Also we have tested that higher value implies higher GW amplitude and lower GW frequency.

Here we can see how the different GW generation mechanisms dominate for different values of h (sound waves, turbulence and collisions). The filled regions are delimited by sensitivity curves of four different eLISA construction designs (now called LISA again).



Of course, we have also tested some other extension of the Standard Model, taking the values where each spectrum reaches its maximum (peak frequency f_p and $\Omega_p \equiv \Omega(f_p)$) and covering a large range of internal parameters (like h in the case of boson/scalar sector coupled to the Higgs)