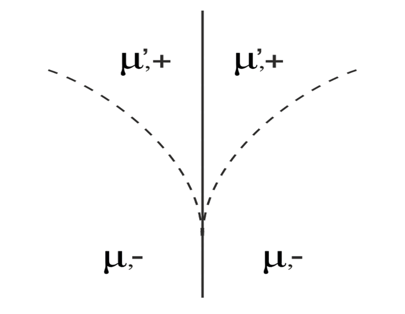

Ceci n'est pas un diagramme de Feynman

Vacuum thin shells in Einstein-Gauss-Bonnet braneworld cosmology

(and a classical mechanism to obtain false vacuum solutions)

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We construct new solutions of the Einstein-Gauss-Bonnet field equations in an isotropic Shiromizu-Maeda-Sasaki brane-world setting which represent a couple of Z2-symmetric vacuum thin shells splitting from the central brane, and explore the main properties of the dynamics of the system. The matching of the separating vacuum shells with the brane-world is as smooth as possible and all matter fields are restricted to the brane. We prove the existence of these solutions, derive the criteria for their existence, analyse some fundamental aspects or their evolution and demonstrate the possibility of constructing cosmological examples that exhibit this feature at early times. We also comment on the possible implications for cosmology and the relation of this system with the thermodynamic instability of highly symmetric vacuum solutions of Lovelock theory.

These are five-dimensional solutions of Einstein-Gauss-Bonnet gravity

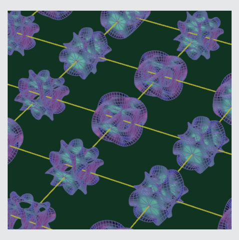
Why considering extra dimensions?

- String/M theory requires the existence of 6 or 7 extra space dimensions
- It makes sense to generalize the established theories to more general dimensionalities
- Also for classical reasons: how unique GR is?
- Lovelock gravity is the most natural and non-trivial generalization of GR for an arbitrary number of space dimensions
- And it is also a classical limit of (certain) string theory

And why can't we see them? Two kinds of explanations...

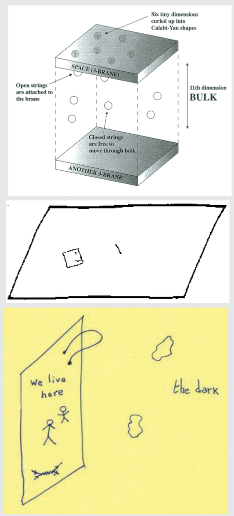
1) Compactification:

- Most common answer (à la Kaluza-Klein): they are compact and very tiny
- We can think that at each point of spacetime there exists an "internal space"
- In string theory, this space is thought to be a 6-dimensional Calabi-Yau manifold



2) Braneworld:

- Horava-Witten: certain string theory has as a limit another theory in R^10xS^1/Z_2 where two 3-branes live
- Inside each brane, the other 6 dimensions are Calabi-Yau compactified. The 11th dimension can be large
- Based on those ideas, Randall and Sundrum created a phenomenological model: within certain limits, our universe might resemble a 4-d manifold (the brane) within a 5-d space-time (the bulk)
- Another answer: our world is confined to a 4-d space-time, which is a submanifold of a higher dimensional space-time (à la Abbot)
- All standard model fields live within one of the branes (there might be several), with the exception of gravity
- Indeed, gravity is what keeps all matter fields confined within the brane-world



Braneworld cosmology

- The Randall-Sundrum model can be adapted to cosmology: the braneworld is a dynamical surface whose intrinsic geometry is Friedmann-Robertson-Walker
- There is a single braneworld at the center of a Z_2-symmetric bulk: the whole 5-d spacetime must be a solution of 5-d general relativity
- The matter content of the braneworld is the one of the standard cosmological model, without dark energy: it is a **thin shell**
- A cosmological constant in the bulk and a brane tension must be imposed
- In order to recover standard cosmology fine tuning is required: the gravitational coupling constant and the 4-d cosmological constant are derived from the parameters of the model
- In the low energy (or large scale parameter) limit, 4-d Einstein gravity is recovered. Nevertheless, there might be measurable deviations from Newton's law at (not-so) small scale...
- Effective field equations on the brane can be written as (ask for details...):

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + \kappa T_{\mu\nu} + \frac{\kappa}{\lambda} S_{\mu\nu} - \epsilon_{\mu\nu} + 4\frac{\kappa}{\lambda} F_{\mu\nu}$$

Lovelock gravity:

Arguably, the most natural higher-dimensional generalization of GR

- It is derived by solving the following problem: **find the most general second order symmetric tensor built solely from the metric and its first two derivatives such that it has an automatically vanishing divergence**
- If we call this tensor A, Lovelock gravity is the theory whose field equations are: $A_{\mu\nu} = \kappa T_{\mu\nu}$
- It turns out that A depends on the dimensionality of spacetime, but it is exactly the Einstein tensor plus a cosmological constant term if spacetime has 4 dimensions
- For 5 or 6 dimensions, the left hand side of the field equations acquires a term that depends quadratically on the curvature, we call it H

$$A_{\mu\nu} = \lambda g_{\mu\nu} + G_{\mu\nu} + \alpha H_{\mu\nu}$$

- This theory can be derived minimizing the following action

$$I = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}(R - 2\Lambda + \alpha L_{GB}) \qquad L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$$

where the Gauss-Bonnet topological invariant (in 4d) can be recognized. This particular case of Lovelock theory is the so-called **Einstein-Gauss-Bonnet gravity**

A simple and usual setting

A 5-dimensional spacetime foliated by 3-dimensional constant curvature manifolds

There is a Birkhoff theorem (Zegers, JMP 2005), so every vacuum region has a metric

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Sigma_k^2$$

$$f(r) = k + \frac{r^2}{4\alpha}\left(1 + \xi\sqrt{1 + \frac{4}{3}\alpha\Lambda + \frac{\alpha\mu}{r^4}}\right)$$

Xi is +1 or -1. The first option is called the "stringy branch", while the second one is the "gr branch"

- The gr-branch has the Schwarzschild-Tangherlini solution as a limit, the stringy one has no small alpha limit
- Both branches are asymptotically dS or AdS (so for a given Lambda and alpha there are two different effective cosmological constants)
- If Lambda=0, the gr-branch is asymptotically flat, but the stringy branch is not

We assume: $\alpha > 0 \qquad 1 + \frac{4}{3}\alpha\Lambda > 0$

Thin shells in EGB gravity

There is a junction condition for a source concentrated on a hypersurface (Davis, PRD 2003 - Gravanis-Willison, PLB 2003)

$$[K^a_b]_\pm - \delta^a_b[K]_\pm + 2\alpha(3[J^a_b]_\pm - \delta^a_b[J]_\pm - 2P^a_{cbd}[K^{cd}]_\pm) = -\kappa_5^2 S^a_b$$

These equations can have a non-trivial solution even if S=0

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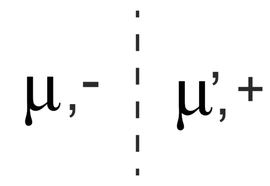
A vacuum thin shell

Garraffo et. al. (JMP 2007) studied the dynamics of vacuum thin shells in isotropic spacetimes

If the orientations of the bulk are standard, then the shell matches two different branches → A "false vacuum" bubble



- The "false vacuum bubbles" can not be stably static nor oscillatory
- Also, the masses of the bulk regions can not be equal
- Two gr-branches can only be glued if alpha<0 and the bulk regions must have the wormhole orientation

The dynamics is described by:

$$\dot{a}^2 + V_{vac}(a, \xi_L A^{1/2}, \xi_R A'^{1/2}) = 0 \qquad A(a) = 1 + \frac{4}{3}\Lambda\alpha + \frac{\alpha\mu}{a^4}$$

And a non-vacuum thin shell

A brane-world in a Z2-symmetric bulk has the equation of motion

$$\dot{a}^2 + V_{GB(+)}(a, \xi A^{3/2}, P^2) = 0$$

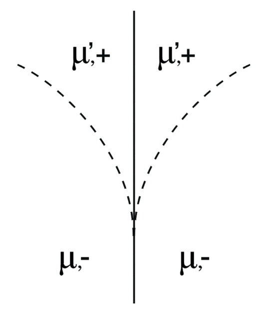
Where the effective potential is written in terms of the function A(a) described before and the function P(a) defined by:

$$P(a)^2 = \frac{\kappa_5^4}{256\alpha^2}\left(\frac{\rho_0}{a^{3\gamma}} + \sigma\right)^2$$

- With some fine tuning of the involving constants, standard cosmology can be recovered in the large "a" limit
- Both the 4-d gravitational coupling constant and cosmological constant appear as functions of the parameters of this construction (hence the fine tuning)
- For small "a", equations are completely different than the standard picture (but there is a big bang)

A vacuum thin shell (softly) emanating from the braneworld

Let us consider a situation like the one illustrated in the figure, where the separation is as smooth as possible: there is continuity of the scale factor of the brane and its first time derivative



- The properties of vacuum thin shells imply that the spacetime between the separating shells, if possible, should be stringy
- The continuity condition determines the mass parameter of the intermediate region
- The possibility of this construction is determined by the relative acceleration of the resulting shells, and a relatively simple criterion can be obtained

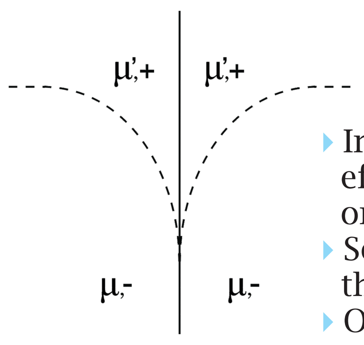
$$x_\infty^{-2/3}\left(1 - \frac{p(a_s)}{\sigma}\right)\left(1 + \frac{\rho(a_s)}{\sigma}\right)^{1/3} > y_s^{-2/3}(x_s)\xi(x_s)$$

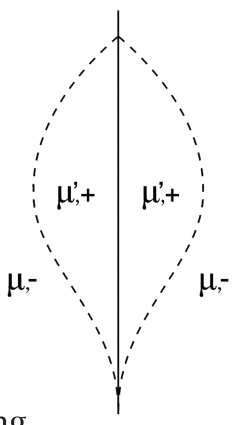
- If satisfied for certain parameters, matter-energy model and scale factor, then the splitting solution is possible (ask for details...)
- This construction for a radiation-dominated braneworld might be possible if "a_s" lies within a certain interval, and provided the "dark radiation" density is large enough in relation to certain combination of the parameters

Can this apply to our universe? (Final outcome of the evolution)

- After the splitting of the vacuum shells, both the Friedmann equations and "effective gravitational coupling constant" change
- Anyway, this can not happen if we are near the "standard model regime", for a radiation-dominated braneworld it can only take place at early (but not arbitrarily early) times
- It can also take place at (arbitrarily) early times if the matter-energy content of the brane-world had non-positive pressure (inflation scenarios)

The vacuum shells might end up colliding with the central shell in a relatively "short" period of time, resulting in the same initial bulk spacetime





- In this case, the parameters can be chosen so that the effective Friedmann equations tend to the standard ones at large scale factor
- So this might apply to our universe without compromising the empirical milestones of the standard model
- Or they might expand indefinitely resulting in a stringy bulk

Another kind of instability?

In recent years there have been different stability analysis regarding solutions of Lovelock theory

- As a higher-order in curvature theory there are in general several distinct vacuum solutions for a given group of isometries and asymptotic structure
- Through semiclassical arguments a thermodynamic instability suggesting spontaneous bubble nucleation and subsequent classical expansion has been studied (Camanho et. al, PRD 2014).
- Through this mechanism a "true vacuum" supersedes a "false vacuum" solution and it can even change the asymptotic structure!
- This stability analysis adds to dynamical stability and it might be more restrictive than the continuity of the map from initial data to solutions (although this is far from obvious)
- The stability against separation of this work can be regarded as another stability analysis for Lovelock gravity
- A generalisation to other settings and arbitrary dimensions, and a comparison with other stability analysis is a matter of future research
- In any case, the present work establishes **a purely classical mechanism to generate a false vacuum bubble** in this context

Final remarks

- We showed that we can smoothly glue vacuum thin shells to non-vacuum thin shells at certain points of the latter's evolution
- This exotic solutions might also signal something pathological regarding thin shells in Lovelock theory
- Similar solutions in GR are possible (Ramirez, CQG 2015), but they always involve separating matter-energy fields. Their strangeness can be understood as a consequence of the neglected matter-energy degrees of freedom. This is not the case
- Also, a formal proof that thin shells in Lovelock are well-defined (whether they make sense as weak solutions) is still lacking